Exercises in Algebraic Topology (master)

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Exercise sheet no 3 for the 22nd and 29th of April 2015

9 (Spheres?)

a) Can you express $H_*(\mathbb{R}P^2\backslash\text{pt})$ as the homology groups of a sphere for a nice point pt $\in \mathbb{R}P^2$?

b) What about $H_*(\mathbb{R}^n\backslash\{0\})$ and $H_*(\mathbb{C}P^2\backslash\text{pt})$?

10 (Wishful thinking?)

Let $n \geqslant 0$ be any natural number. Can you find a pair of spaces (X, A) such that A is not the empty set and

$$H_0(X, A) \cong H_0(X\backslash A) \cong \mathbb{Z}^n?$$

11 (Too ugly?)

What can you say about $H_1(\mathbb{R}, \mathbb{Q})$? Is it free abelian? Does it have torsion?

12 (Linear algebra)

Compare the homology groups of $GL_n(\mathbb{R})$ and $O(n)$. What about $GL_n(\mathbb{C})$ and $U(n)$?