

Exercises in Algebraic Topology (master)

Prof. Dr. Birgit Richter

Summer term 2015

Exercise sheet no 12

For the exercise classes on the 24th of June and 1st of July 2015

45 (Exactness of direct limits) Prove the remaining two bits that establish that direct limits map short exact sequences of directed systems of R -modules to short exact sequences of R -modules (proof of lemma 3.7).

46 (Compact support)

a) If X is a path-connected, non-compact space, what is $H_c^0(X)$?

b) Prove a version of suspension for cohomology with compact support, i. e., show that $H_c^n(X \times \mathbb{R}) \cong H_c^{n-1}(X)$ for all $n \geq 1$.

47 (Degree) Let M be a connected oriented compact m -manifold. We can assign a degree map to M by sending an $f \in [M, \mathbb{S}^m]$ to its mapping degree. Is that map always surjective as a map to the integers?

48 (3-manifolds) Let M be a compact connected 3-manifold without boundary. Its first homology group is a finitely generated abelian group and is hence of the form

$$H_1(M) \cong \mathbb{Z}^n \oplus T$$

where T denotes the torsion part of $H_1(M)$.

a) Determine $H_2(M)$ if M is orientable.

b) Does $\pi_1(M)$ determine $H_*(M)$ in this case?

c) What happens if we drop the assumption that M is orientable? What do you get for $H_2(M)$?