Exercises in Algebraic Topology (master)

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Exercise sheet no 8 due: 7th of June 2013

29 (Products of Moore spaces)

Let $M(\mathbb{Z}/p\mathbb{Z}; n)$ and $M(\mathbb{Z}/q\mathbb{Z}, m)$ be two Moore spaces with p, q prime and $n, m \geqslant 1$. What are the homology groups of the product $M(\mathbb{Z}/p\mathbb{Z}; n) \times M(\mathbb{Z}/q\mathbb{Z}, m)$? Make sure to cover all possible cases.

30 (Field coefficients)

Let k be a field and let X and Y be arbitrary topological spaces. Show that for all $n \geqslant 0$

$$\bigoplus_{p+q=n} H_p(X; k) \otimes_k H_q(Y; k) \cong H_n(X \times Y; k).$$

Here, $H_p(X; k) \otimes_k H_q(Y; k)$ denotes the tensor product over k of the k-vector spaces $H_p(X; k)$ and $H_q(Y; k)$.

31 (Euler characteristic of products)

Let X and Y be topological spaces whose homology groups are finitely generated and are non-trivial in finitely many degrees. Prove the multiplicity of the Euler characteristic, *i.e.*, show that

$$\chi(X \times Y) = \chi(X)\chi(Y).$$

32 (non-natural)

Give an explicit example for the fact that the splitting in the topological Künneth formula is not natural.