Exercises in Algebraic Topology (master)

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Exercise sheet no 7 due: 31st of May 2013

25 (Moore spaces) Let G be an arbitrary finitely generated abelian group.

a) Construct a CW space $M(G,n)$ whose reduced homology is concentrated in degree n with $\tilde{H}_n(M(G,n)) \cong G$. Can you extend the construction to the case where G is not necessarily finitely generated?

Such a space is called a *Moore space of type* (G,n).

b) How does $M(G,n)$ look like if G is a finite cyclic group?

c) Do you recognize $M(\mathbb{Z}/2\mathbb{Z}, 1)$?

d) Let $A_1, A_2, \ldots$ be a sequence of abelian groups. Construct a space X with $\tilde{H}_i(X) \cong A_i$.

26 (Künneth formula, special case) You've seen a proof of the (algebraic, homological) Künneth formula.

a) Give a simplified proof in the special case where the second chain complex is just C_G, *i.e.*, $(C_G)_n = G$ for $n = 0$ and $(C_G)_m = 0$ otherwise.

b) What is a non-natural splitting of the short exact sequence

$$0 \to H_n(C_*) \otimes G \longrightarrow H_n(C_* \otimes G) \longrightarrow \mathrm{Tor}(H_{n-1}(C_*), G) \to 0$$

in this case?

27 (right exactness)

a) Prove the right exactness of the tensor product.

b) Is the abelian group $\mathbb{Q}$ free? Is the functor $A \mapsto A \otimes \mathbb{Q}$ exact?

28 (boomerang?) Let A be a finitely generated abelian torsion group. Can you identify $\mathrm{Hom}(A, \mathbb{Q}/\mathbb{Z})$ and/or $\mathrm{Tor}(A, \mathbb{Q}/\mathbb{Z})$ with A?