Exercises in Algebraic Topology (master)

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Exercise sheet no 6 due: 17th of May 2013

21 (Standard examples) Write down explicit CW models for the 2-torus, for the Klein bottle and for the real and complex projective spaces.

22 (Rotten apple?) Is it possible to turn the Hawaiian earring into a CW complex? What about the cone on the Hawaiian earring?

23 (Euler characteristic) Let X be a finite CW complex. The *Euler characteristic* of X, $\chi(X)$, is then defined as

$$\chi(X) := \sum_{n \geqslant 0} (-1)^n \mathrm{rk}(H_n(X; \mathbb{Z})).$$

Here, rk denotes the rank of a finitely generated abelian group, *i.e.*, the number of its free summands.

a) Why is $\chi(X)$ well-defined?

b) What is $\chi(X)$ for a torus, a sphere or a general oriented compact closed surface of genus g, F_g?

c) Let $c_n(X)$ denote the number of n-cells of X. Prove that

$$\chi(X) = \sum_{n \geqslant 0} (-1)^n c_n(X).$$

d) For finite CW complexes X and Y, show that $\chi(X \times Y) = \chi(X)\chi(Y)$.

e) For a finite CW complex X and $p\colon \tilde{X} \to X$ an n-sheeted covering space, show that $\chi(\tilde{X}) = n\chi(X)$.

24 (Gluing) Let

$$W = \{(x, y, z) | |x|, |y|, |z| \leqslant 1\}$$

be the standard 3-cube. We identify opposite faces of W by gluing them after a 90 degree anti-clockwise turn, *i.e.*, $(x, y, 1) \sim (-y, x, -1)$, $(x, 1, z) \sim (-z, -1, x)$ and $(1, y, z) \sim (-1, -z, y)$.

a) Draw that!

b) What are the homology groups of $W/\sim$?