

## Exercises in Algebraic Topology (master)

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Summer term 2013

### Exercise sheet no 3

due: 26th of April 2013

#### 9 (Spheres?)

- a) Can you express  $H_*(\mathbb{R}P^2 \setminus \text{pt})$  as the homology groups of a sphere for a nice point  $\text{pt} \in \mathbb{R}P^2$ ?
- b) What about  $H_*(\mathbb{R}^n \setminus \{0\})$  and  $H_*(\mathbb{C}P^2 \setminus \text{pt})$ ?

#### 10 (Wishful thinking?)

Let  $n \geq 0$  be any natural number. Can you find a pair of spaces  $(X, A)$  such that  $A$  is not the empty set and

$$H_0(X, A) \cong H_0(X \setminus A) \cong \mathbb{Z}^n?$$

#### 11 (Too ugly?)

What can you say about  $H_1(\mathbb{R}, \mathbb{Q})$ ? Is it free abelian? Does it have torsion?

#### 12 (Linear algebra again)

Compare the homology groups of  $GL_n(\mathbb{R})$  and  $O(n)$ . What about  $GL_n(\mathbb{C})$  and  $U(n)$ ?