Exercises in Algebraic Topology (master)

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Exercise sheet no 11 due: 28th of June 2013

41 (exactness of direct limits) Prove the remaining two bits that establish that direct limits map short exact sequences of directed systems of R-modules to short exact sequences of R-modules (proof of lemma 3.7).

42 (compact support)

a) If X is a path-connected, non-compact space, what is $H^0_c(X)$?

b) Prove a version of suspension for cohomology with compact support, i. e., show that $H^n_c(X \times \mathbb{R}) \cong H^{n-1}_c(X)$ for all $n \geqslant 1$.

43 (degree) Let M be a connected oriented compact m-manifold. We can assign a degree map to M by sending an $f \in [M, \mathbb{S}^m]$ to its mapping degree. Is that map always surjective as a map to the integers?

44 (3-manifolds) Let M be a compact connected 3-manifold without boundary. Its first homology group is a finitely generated abelian group and is hence of the form

$$H_1(M) \cong \mathbb{Z}^n \oplus T$$

where T denotes the torsion part of $H_1(M)$.

a) Determine $H_2(M)$ if M is orientable.

b) Does $\pi_1(M)$ determine $H_*(M)$ in this case?

c) What happens if we drop the assumption that M is orientable? What do you get for $H_2(M)$?