Fachbereich Mathematik,
Universität Hamburg
WS 23/24

PD Dr. Max Pitz
Thilo Krill

Infinite graphs

Sheet 13

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1. Show that the graph H from Figure 9.1.2 fails to be topologically minor ubiquitous.

2. Let $\mathcal{A}$ be a finite set of disjoint rays in a graph G. Show that two rays $R, S \in \mathcal{A}$ are in the same component of $K(\mathcal{A})$ if and only if they belong to the same end of G.

3. Let $\mathcal{A}$ be a finite set of disjoint rays in a graph G with ray-graph $K = K(\mathcal{A})$. Let X be a finite set of vertices of G. Let $\mathcal{R} = (R_1, \dots, R_n)$ and $\mathcal{S} = (S_1, \dots, S_n)$ be sequences of disjoint rays in $\mathcal{A}$. If one can win the $\mathcal{R} - \mathcal{S}$ pebble-pushing game on K, then there is an $\mathcal{R} - \mathcal{S}$ linkage $\mathcal{P}$ after X linking R_i to S_i for all $i \leq n$.

4. Show that there does not exist a locally finite connected graph which contains a copy of every locally finite connected graph as a subgraph.

5. Show that there exists a locally finite connected graph which contains a copy of every locally finite connected graph as a topological minor.

6. The following concerns the proof of Theorem 10.1.7:

- Give an example where a cut-faithful decomposition into countable fragments fails to be degree-faithful.
- Fill in the missing details that every graph has a decomposition into countable, cut- and degree-faithful fragments.

7. Prove Theorem 10.1.7(1) and (2) in the countable case.

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Hints

1. –

2. –

3. –

4. Suppose that G is such a locally finite graph. Construct a locally finite connected graph H whose vertex degrees 'grow too fast' for any embedding of H in G.

5. –

6. –

7. –