Exercise Sheet 12 for Infinite Graph Theory, WS 2019/20
(to be discussed on 20. January 2020)

1.$^{-}$ Verify that the class of finite graphs without induced $\overline{K^r}$ is an essentially countable amalgamation class.

2. Show that every essentially countable class $\mathcal{C}$ of L-structures is the age of a countable L-structure if and only if $\mathcal{C}$ it is closed under isomorphisms, has the joint embedding property and the hereditary property.

3.$^{-}$ Prove that every 2-edge-connected graph has a collection $\mathcal{C}$ of cycles such that every edge of G is in at least one and at most countably many cycles in $\mathcal{C}$.

4. Prove that in order to establish Nash-Williams' orientation theorem for arbitrary graphs it suffices to prove the statement for countable graphs.

5. Prove Lemma 2 from the lectures on Laviolette's cut-faithful decomposition theorem.

6. Show that a graph G can be decomposed into cycles and 2-way infinite tours if and only if it has no vertex of odd degree.

7.$^{+}$ Show that a graph G is decomposable into 2-way infinite tours if and only if it has no vertex of odd degree and no finite non-trivial component.

Optional:

8.$^{++}$ Can you find a characterisation for a graph to decomposable into double rays (plus maybe cycles)?

9.$^{++}$ Improve Q3 and show that every 2-edge-connected graph has a collection $\mathcal{C}$ of cycles such that every edge of G is in at least one and at most finitely many cycles in $\mathcal{C}$. Can you do "in at most 6 cycles"?

Hinweise

1.$^{-}$

2. Construct the desired countable structure as the union of a suitable chain of elements in $\mathcal{C}$.

3.$^{-}$ Laviolette.

4. Laviolette.

5. Similar to Lemma 1.

6. Laviolette.

7.$^{+}$ First prove the countable case. To reduce to the countable case, generalise Laviolette's construction by adding a suitable condition to the construction of the G_α, similar to (4), but not about paths in $\mathcal{P}(v, w)$ for distinct $v \neq w$, but instead about vertices in $N_G(v)$ for $v \in V(G)$.

8.$^{++}$

9.$^{++}$