Stochastische dynamische Systeme und Ergodentheorie

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Jordan normal form for random matrices

Our aim is to classify (products of) random matrices with respect to random similarity, to characterize the equivalence classes by invariants, and to determine the "simplest possible" member of a class (Jordan normal form). The difficulties become already apparent in the "trivial case" of dimension 1, where the problem reduces to solving a cohomological equation. We give an explicit treatment of the case of dimension 2 and finally present our general result: Every random matrix is similar to an upper block-triangular matrix whose diagonal blocks are irreducible and "block-conformal" (i.e. are random permutations of conformal sub-blocks). This is joint work with Nguyen Dinh Cong (Hanoi) and Valery Oseledets (Moscow).

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Limit Theorems for Higher Windings of Brownian Paths in Riemannian Manifolds

The asymptotics of windings of Brownian paths on Riemannian manifolds — represented as elements of the 1st homology group — drew a lot of attention in the literature and a lot of results on them has been obtained (by Spitzer, McKean, Yor, Kozlov,...). Much less is known about the properties of the random elements of the *fundamental group* these paths define. We apporach this question by studying the nilpotent approximation to the fundamental group (of which H_1 is the first one) using the iterated integrals formalism of Chen. It turns out that (in the case of compact Riemannian manifold) the corresponding higher windings converge to the heat kernels of hypoelliptic operators associated to a subRiemannian structure, studied by Gaveau and others. Some applications are presented, i.a. the asymptotics of finite-type invariants for random braids.

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Die Hausdorff-Dimension konformer Repeller unter zufälligen Störungen

In diesem Vortrag werde ich über ein gemeinsames Projekt mit Gunter Ochs (Bremen) berichten, dessen Ziel es ist, die folgende Aussage rigoros zu machen: Ist ein zufälliges dynamisches System nahe an einer deterministischen expandierenden Abbildung auf einem konformen Repeller, so sind die entsprechenden Hausdorff-Dimensionen nahe. Wesentlicher Bestandteil wird hierbei eine Verallgemeinerung der Bowen-Ruelle Formel auf das zufällige Set-up sein.

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Stochastic Equations in Fluid Dynamics

Stochastic partial differential equations of Navier Stokes and Euler type have been investigated in recent years by many authors, both as more realistic models in certain situations, and with the aim to prove results with are not accessible in the deterministic setting. Some of the recent advances in the field are overviewed. These equations have a number of interesting dynamical properties, including a stochastic form of dissipativity, the existence of a compact random attractor (a notion now of interest in other fields, but originated during the investigation of these equations), its finite Hausdorff dimension, and other indicators of finite dimensionality of the asymptotic dynamic as the property of determining modes. Ergodic properties have been proved (in contrast to the deterministic models), like uniqueness of the invariant measure and the ergodic theorem, including recently an indication of the spectral gap condition. Most of these results have been obtained for 2-dimensional equations, but some properties hold true also in the 3-dimensional case (where the well posedness of the equations is an open problem), like a form of irreducibility, which can be proved in spite of the lack of continuous dependence on data. The backward Kolmogorov equation in Hilbert spaces associated to the stochastic equation of Navier Stokes type, has been solved by a completely analytic approach, in a particular situation; this opens the possibility to prove results for the stochastic Navier Stokes equation as a by-product of results for the associated Kolmogorov equation.

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Transversal Markov Processes for Dynamical Systems and their Applications to Limit Theorems

Let $(X, \mathcal{F}, \mathcal{P}, \mathcal{T})$ be a probabilistic dynamical system where T is an automorphism (a measure preserving transformation) of a probability space $(X, \mathcal{F}, \mathcal{P})$ and let U be a unitary operator in $L_2(X) = L_2(X, \mathcal{F}, \mathcal{P})$ defined by $Uf = f \circ T$. Let A_s and A_u be generators of symmetric Markov transition semigroups $\{P_s(t) = \exp(-A_s t), t \in R^+\}, \{P_u(t) = \exp(-A_u t), t \in R^+\}$ (subscripts are abbreviations for 'stable' and 'unstable') acting in L_2. A_s and A_u are called transversal Markov generator for T if for some real number $\theta \in (1, \infty)$.

$$U^{-1} P_s(t) U = P_s(\theta^{-1} t), \quad U^{-1} P_u(t) U = P_u(\theta t)$$

for $t \in \ R^+$, or, equivalently,

$$U^{-1} A_s U = \theta^{-1} A_s, \quad U^{-1} A_u U = \theta A_u.$$

Generalizing a construction introduced in a earlier paper of the author (where an automorphism supplied with only one expanding Markov generator A_u was considered) we represent X as a state space of a nonhomogeneous Markov process $\{\xi(t), t \in R\}$ governed by generators A_s and A_u on $(-\infty, 0]$ and $[0, \infty)$, correspondingly, and such that $\xi(0)$ is distributed according to the measure P. Thus the sample paths of this process become the points of a probability space $(X', \mathcal{F}', \mathcal{P}')$. Then $\pi : X' \ni \xi(\cdot) \mapsto \xi(0) \in X$ is a measure preserving map. One can also define an automorphism T' of $(X', \mathcal{F}', \mathcal{P}')$. Moreover, $(X', \mathcal{F}', \mathcal{P}', \mathcal{T}')$ is an extension of $(X, \mathcal{F}, \mathcal{P}, \mathcal{T})$ that is $\pi\ T' = T\ \pi$. The advantage of considering such extension is that $(X', \mathcal{F}', \mathcal{P}', \mathcal{T}')$ has a rich structure of various $\sigma-$fields what could be not so for $(X, \mathcal{F}, \mathcal{P}, \mathcal{T})$. Under the assumptions that A_s and A_u commute and that their sum $A_s + A_u$ is bounded below apart from zero we establish very strong decay to zero of the maximal correlation coefficient between the natural $\sigma-$fields of 'the Future' and 'the Past' in $(X', \mathcal{F}', \mathcal{P}')$.

The following result is proved as an application of this construction. Let $f \in L_2$ be such that $\int f(x)P(dx) = 0$ and

$$\sum_{k \geq 0} (|P_s(\theta^{-k})f - f|_2 + |P_u(\theta^{-k})f - f|_2) \leq \infty.$$

Then the sequence $\{f \circ T^k, \quad k \in Z\}$ satisfies the Functional Central Limit Theorem and also Strassen's form of the Law of the Iterated Logarithm.
As an example we consider hyperbolic toral automorphisms where the construction of transversal Markov generators is rather interesting. This construction generalizes another paper of the author and presents unbounded second oder difference operators. Corresponding Markov processes evolute with jumps along, respectively, stable and unstable directions. Allowed jumps make a group which is called homoclinic. This special choice of jumps plays a crucial role in the proof of semiboundedness property. Other related examples and problems will be also discussed.

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Stochastic rotation numbers

The Lyapunov numbers of random dynamical systems generated by the linearizations of stochastic differential equations provide a finite set of real numbers describing the asymptotic exponential growth rates of their trajectories. In a similar way, the asymptotic rates of rotation of a vector moved by the system within a moved plane can take only a finite number of possible real values, the *rotation numbers.* Behind this statement is a multiplicative ergodic theorem for rotation numbers.

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Explizite Lyapunovexponenten für den verrauschten gedämpften harmonischen Oszillator

Die Lyapunovexponenten der Linearisierung

$$dv_t \;=\; \begin{pmatrix} 0 & 1 \\ -1 & \beta \end{pmatrix} v_t \; dt + \begin{pmatrix} 0 & 0 \\ \sigma & 0 \end{pmatrix} v_t \circ dW_t$$

des zweidimensionalen verrauschten Duffing-van der Pol-Oszillators sind Schlüsselgrößen für die Untersuchung der stochastischen Hopfbifurkation dieses Systems. Es wird eine einfache Gleichung hergeleitet, die die Lyapunovexponenten dieses Systems als Funktion des zweiten Moments des invarianten Maßes einer assoziierten Diffusion ausdrückt. Damit lassen sich explizite Formeln für die Lyapunovexponenten mittels hypergeometrischer Funktionen in den Parametern β und σ angeben. Ferner führt diese Darstellung sowohl zu vollständigen und expliziten asymptotischen Entwicklungen als auch zu Aussagen über einige globale Eigenschaften der Lyapunovexponenten als Funktionen des Dämpfungsparameters β und des Rauschparameters σ.

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Diskretisierung von zufälligen dynamischen Systemen

Die Untersuchung von Attraktoren für dynamische Systeme, die durch stochastische bzw. zufällige Differentialgleichungen generiert werden, hat in den letzten Jahren eine wichtige Rolle gespielt. Um Aussagen über die Struktur dieser zufälligen Attraktoren zu gewinnen, erscheinen numerische

Simulationen unumgänglich.
Es soll ein zufälliges dynamisches System betrachtet werden, welches durch nichtäquidistante Diskretisierung einer zufälligen Differentialgleichung entsteht. Es wird nachgewiesen, daß für eine gewisse Klasse von zufälligen Differentialgleichungen dieses diskrete zufällige dynamische System ebenfalls Attraktoren besitzt. Weiterhin werden Beziehungen dieser Attraktoren zu den Attraktoren der Ausgangsgleichungen diskutiert.

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One dimensional bifurcation of random dynamical systems

We consider families of random dynamical systems induced by parametrized one dimensional stochastic differential equations. We give necessary and sufficient conditions on the invariant measures of the associated Markov semigroups which ensure a stochastic bifurcation. This leads to sufficient conditions on drift and diffusion coefficients for a stochastic pitchfork and transcritical bifurcations of the family of random dynamical systems.

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Locally contractive iterated function systems

We present a new method for proving the convergence of iterated function systems of the form $F_n(x) = f_1(f_2(\cdots(f_n(x))\cdots))$, where the f_i are i.i.d. random functions on a convex subset of $\mathcal{R}^d$. This method is designed to work in cases where more conventional methods break down, when the random function is not everywhere contractive on average, but where there is some region of contractivity. In some cases we also obtain sharp bounds on the rate of convergence. Some applications are the random logistic maps described by Letac and Chamayou, and the self-modifying process "Zeno's Walk".

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On Gaussian character of almost sure convergence

We establish functional type inequalities linking the regularity properties of sequences of operators $S = (S_n)$ acting on L^2-spaces, with those of the canonical Gaussian process restricted on associated subsets of L^2 defined by $(S_n(f))$, $f \in L^2$. These inequalities allow to easily deduce as corollaries famous Bourgain's entropy criterias in the theory of almost everywhere convergence. These also provide a better understanding of the role of the Gaussian processes in the study of phenomenon of almost everywhere convergence.

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Some Uniform Ergodic Inequalities in the non-measurable Case

Uniform and non-measurable versions of some classical ergodic inequalities (all of them going back to the Hopf-Yosida-Kakutani maximal ergodic theorem) are established. Usually, uniformity involves non-measurable suprema and all the technical difficulties arising from this. In the present paper, a simplification is achieved by extending the given operator (a positive $\mathcal{L}_\infty$-contraction) to

the class of all (i.e., not necessarily measurable) functions on the underlying measure space. This is not only leading to technical improvements and clarifications of the proofs, but also to remarkable generalizations of known results. In particular, it turns out that the “operator” under consideration need not even be an extension of a $\mathcal{L}_\infty$-contraction, but has only to fulfill some mild conditions such as positivity, superadditivity and a certain contractivity property involving upper integrals.