MÜNCHENER STOCHASTIK-TAGE 1998 Sekt.-Nr.:10

Stochastische Analysis (einschließlich Stochastik der Finanzmärkte)

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A Central Limit Theorem for Schroedinger Operators with Random Potentials

In a wide variety of models in quantum mechanics a sequence (X_n) of pairwise independent identically distributed random variables is given and one considers potentials V_N of the type $V_N(x) = (1/N)\sum_{n=1}^{N} W(x - X_n)$. Here W is some fixed potential which vanishes outside some small domain and N is a large number. I shall give conditions which are sufficient in order that the operators $-\Delta + V_N$ converge a.s. to the same limit as N tends to infinity, discuss the rate of convergence and spectral properties of the operators.

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Stochastic differential equations on infinite products of compact manifolds and applications to statistical mechanics

Let M be a compact Riemannian manifold and $\mathbf{Z}^d$ be the $d-$dimensional integer lattice. We consider a stochastic differential equation on $M^{\mathbf{Z}^d}$ and prove existence and uniqueness theorem for it. This result gives us the possibility to construct stochastic dynamics associated with Gibbs measures on $M^{\mathbf{Z}^d}$ and to give a probabilistic representation of the corresponding Feller semigroup. We prove the uniqueness of the dynamics and the equivalence of its L_2-ergodicity and the extremality of the corresponding Gibbs measure.

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Hyperbolic models in finance

Distributional assumptions for the returns of the underlying assets play a key role in valuation theories for derivative securities. Empirical investigations showed that hyperbolic distributions allow an almost perfect fit of return distributions for various assets. This led us to consider a Lévy process (X_t), which we call a hyperbolic Lévy motion, as the process driving asset prices. The corresponding model is given by the equation

$$dS_t = S_{t^-}\left[\mu dt + \sigma dX_t + \left(e^{\sigma\Delta X_t} - 1 - \sigma\Delta X_t\right)\right].$$

The hyperbolic Lévy motion is a purely discontinuous process and thus allows modeling of the microstructure of empirical asset price paths.
We derive an option pricing formula and compare the results to option quotes of the exchange. As one consequence we see a reduction of the smile effect. We discuss a statistical martingale measure which not only improves option pricing but at the same time provides an excellent measure for market risk.
We also present a new class of bond price models that can be driven by a wide range of Lévy processes. This is done in the context of martingale modeling.

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Über Dirichletfunktionen der Brownschen Bewegung

Eine meßbare reelle Funktion f heißt Dirichletfunktion der eindimensionalen Brownschen Bewegung $(B, \mathbb{F})$, falls $f(B)$ ein lokaler Dirichletprozeß ist, das heißt, eine Zerlegung

$$f(B) = M + Q \qquad P\text{-f.s.}$$

mit einem lokalen Martingal $(M, \mathbb{F})$ und einem Prozeß von quadratischer Variation Null besitzt. Dabei heißt ein (rechtsstetiger) Prozeß $(Q, \mathbb{F})$ von quadratischer Variation Null, wenn er vermöge einer nichtfallenden Folge (T_n) von Stoppzeiten mit $\lim_{n\to\infty} T_n = +\infty$ P-f.s. reduziert werden kann auf Prozeße $(Z, \mathbb{F})$ mit der Eigenschaft

$$\lim_{\pi} \mathrm{E} \sum_{k=0}^{m-1} (Z_{t_{k+1}} - Z_{t_k})^2 = 0, \tag{1}$$

wobei der Grenzwert über eine geeignete Folge von Zerlegungen π genommen wird. Beziehung (1) is äquivalent zu

$$\lim_{\varepsilon\to 0} \mathrm{E}\frac{1}{\varepsilon} \int_0^\infty (Z_{s+\varepsilon} - Z_s)^2 \, ds = 0. \tag{2}$$

Im Vortrag geben wir eine analytische Charakterisierung von Dirichletfunktionen f einer (reflektierten) Brownschen Bewegung. Als eine Anwendung führt das Ergebnis zu einer expliziten Darstellung beliebiger additiver stetiger lokaler Dirichletprozesse der Brownschen Bewegung als die Summe eines stochastische Integrals nach der Brownschen Bewegung und eines Integrals nach ihrer lokalen Zeit als Funktion des Raumparameters.

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Schwache Ableitungen für Optionswerte in Monte–Carlo–Simulationen

Monte–Carlo–Simulation ist eine Alternative für die Analyse des Wertes einer Option, wenn die Annahmen vereinfachender analytischer Modelle verletzt werden. In unserem Vortrag stellen wir einen Ansatz vor, mit dem die Sensitivitätsanalyse eines Optionspreises mit Hilfe von Simulationen effizient durchgeführt werden kann. Dieser – auf der Theorie der schwachen Ableitung beruhende – Ansatz erlaubt es insbesondere, aus einem simulierten Pfad nicht nur den Wert der Option sondern auch die Sensitivität des Optionswertes bezüglich verschiedener Modellparameter zu ermitteln. Wir diskutieren Anwendungen auf Europäische sowie Amerikanische Optionen. Ferner betten wir unseren Gradientenschätzer in eine stochastische Optimierung ein. Numerische Beispiele illustrieren die Effizienz des Algorithmus. Erweiterungen auf allgemeinere Probleme werden diskutiert.

Gefördert durch das EC–TMR Projekt ALAPEDES unter ERBFMRXCT960074, siehe dazu auch http://www.cs.rug.nl/~rein/alapedes/alapedes.html

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Zinsstrukturmodelle mit expliziten Formeln für Bondpreise

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A non-explosion result for a class of jump processes

We consider Markov jump type processes which are generated by a pseudo differential operator $p(x, D)$. The corressponding symbol $p(x, \xi)$ has the property that for fixed x the function $p(x, \cdot)$ is a continuous negative definite function. Depending on the type of symbol in terms of the ξ-dependence we give maximal growth condition of symbol for large x which implies the existence of a non-exploding solution of the martingale problem. Moreover, the solution is unique provided the symbol is sufficently smooth. This result in particular includes the well-known quadratic growth condition for the diffusion matrix of a diffusion process.

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Hyperbolic Distributions and Option Pricing

In this talk we use the hyperbolic Lévy model proposed by Eberlein and Keller [1] to price Exotic options using the risk-neutral Esscher measure introduced by Gerber and Shiu, see [2]. Furthermore we discuss implications for the hedging of such options.

[1] E.Eberlein and U.Keller: Hyperbolic distributions in finance. *Bernoulli*, 1:281–299, 1995.
[2] H.U. Gerber and E.S.W. Shiu: Actuarial approach to option pricing. Université de Lausanne, ISA, cahier 95.02, 1995.

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Classical gas in Riemannian space

We consider Gibbs measures for continuous classical statistical mechanics where the single particles are considered in a (non-compact) Riemannian manifold X. The interaction is given by a pair potential which fulfills an analogue of the stability and integrability condition. Using Kirkwood-Salzburg equation we obtain existence of this measures in the high temperature regime. In this regime we construct also a cluster expansion and show its convergence. Using the measures discussed we construct representations of the group $\mathrm{Diff}_0(X)$ (compactly supported diffemorphisms of X).

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Replication Costs for American Derivatives with Transaction Costs

Let $N \geq 1$, and let $\mathcal{Z} = \{z_0, z_1, ..., z_N\}$ be a subset in $\mathbf{R}^N$ such that the convex hull of $\mathcal{Z}$ contains the origin as an inner point. Then there are $\hat{p}_0, \hat{p}_1, ..., \hat{p}_N \in (0,1)$ such that $\sum_{i=0}^N \hat{p}_i = 1$ and that $\sum_{i=0}^N \hat{p}_i z_i = 0$. Let $\Omega = \mathcal{Z}^{\mathbf{N}}$. Then Ω is a Polish space. Let $\mathcal{F}$ be the Borel algebra on Ω. Let $Z_k(\omega) = \omega_k$, $k = 1, 2, \ldots$, $\omega = (\omega_1, \omega_2, \ldots) \in \Omega$. Let $\mathcal{F}_{-1} = \mathcal{F}_0 = \{\phi, \Omega\}$, $\mathcal{F}_k = \sigma\{Z_1, Z_2, \ldots, Z_k\}$,

$k = 1, 2, \ldots$. Let P be a probability measure on Ω such that Z_k, $k = 1, 2, \ldots$, are independent identically distributed random variables under P and that $P(Z_1 = z_i) = \hat{p}_i$, $i = 0, 1, \ldots, N$.
Let h be a positive number. We consider h the unit of time. We think of h as a small parameter and will take $h \downarrow 0$ eventually. Let $r \geq 0$ and $\hat{S}_0 = (\hat{S}_0^0, \hat{S}_0^1, \ldots, \hat{S}_0^N) \in \mathbf{R}^{1+N}$. Let $\alpha : \mathbf{R}^{1+N} \to \mathbf{R}^N \otimes \mathbf{R}^N$ be a Lipschitz continuous function. Now let $S(k) = (S^0(k), S'(k))$, $k = 0, 1, 2, \ldots$, be $\mathbf{R}^{1+N}$-valued random variables inductivel defined by

$$S(0) = \hat{S}_0$$

$$S^0(k+1) = (1 + rh)S(k)$$

$$S'(k+1) = (1 + rh)S'(k) + h^{1/2}\alpha(S(k))Z_{k+1}$$

for $k = 0, 1, 2, \ldots$. We assume that there are $1 + N$ kinds of securities and the price process of them is given by $\{S(k)\}_{k=0}^{\infty}$.
Let $\varphi : \mathbf{R}^N \times \mathbf{R}^{1+N} \to [0, \infty)$ be a Lipschitz continuous function such that $\varphi(\cdot, x) : \mathbf{R}^N \to [0, \infty)$ is a convex homogeneous of order one function for each $x \in \mathbf{R}^{1+N}$. We assume that there is no transaction costs for the trade of 0-th security and that the transaction cost for the other N kinds of securities at time k is given by $h^{1/2}\varphi({}^t\alpha(S(k))\xi, S(k))$ when the trade amount is $\xi \in \mathbf{R}^N$.
Let $\mathcal{I}^N$ denotes the set of all $\mathbf{R}^N$-valued $\mathcal{F}_k$-adapted processes. Let $X : \{-1, 0, 1, 2, \ldots\} \times \mathcal{I}^N \times \Omega \to \mathbf{R}^{1+N}$ be inductly given by

$$X(-1, I, \omega) = 0,$$

$$X^0(k, I, \omega) = X^0(k-1, I, \omega) - S^0(k)^{-1}\{I(k) \cdot S'(k) + h^{1/2}\varphi({}^t\alpha(S(k))I(k))\}$$

$$X'(k, I, \omega) = X'(k-1, I, \omega) + I(k, \omega)$$

for $I \in \mathcal{I}^N$ and $k = 0, 1, 2, \ldots$. Here $X(k, I, \omega) = (X^0(k, I, \omega)), X'(k, I, \omega))$. Then, if we take a trading strategy $I \in \mathcal{I}$ and the initial portfolio is $x \in \mathbf{R}^{1+N}$, then the post-trade portfolio at time k is given by $x + X(k, I)$.
For $K \geq 1$ and any $\mathcal{F}_k$-adapted process $Y : \{0, 1, ..., K\} \times \Omega \to \mathbf{R}$, let

$$\begin{aligned} & \mathcal{C}_K(Y) \\ = \ & \{(x^0, I) \in \mathbf{R} \times \mathcal{I}^N; \quad x^0 S^0(k, \omega) + X'(k-1; I) \cdot S'(k) - h^{1/2}\varphi({}^t A(S(k))X'(k-1; I)) \\ & \qquad \geq Y(k), k = 0, \ldots, K\}. \end{aligned}$$

and

$$\pi_{h,K}(Y) = \inf\{x^0 S^0(0);\ (x^0, I) \in \mathcal{C}(Y) \text{ for some } I \in \mathcal{I}^N\}.$$

Then $\pi_{h,K}(Y)$ can be regarded as the super-replication cost or the headging cost for the American derivative such that the maturity is K and the pay-off at time kh is $Y(k)$.
Now let $T > 0$ and $F : [0, T] \times D([0, \infty); \mathbf{R}^{1+N}) \to \mathbf{R}$ be a measurable map satisfying the following assumptions.
(1) $F(t, \cdot)$ is $\sigma\{w(s); 0 \leq s \leq t\}$-measurable for each $t \in [0, T]$.
(2) If $(t, w) \in [0, T] \times C([0, \infty); \mathbf{R}^{1+N})$, $(t_n, w_n) \in [0, T] \times D([0, \infty); \mathbf{R}^{1+N})$, $n = 1, 2, \ldots$ and if $(t_n, w_n) \to (t, w)$ in $[0, T] \times D([0, \infty); \mathbf{R}^{1+N})$, $n \to \infty$, then $F(t_n, w_n) \to F(t, w)$ as $n \to \infty$.
(3) There are $p \in (1, \infty)$ and $C > 0$ such that

$$| F(t, w) | \leq C(1 + \sup_{s \in [0,t]} | w(s) |^p), \qquad (t, w) \in [0, T] \times D([0, \infty); \mathbf{R}^{1+N}).$$

Let $W_h(t, \omega) = S([h^{-1}t], \omega)$, $t \in [0, \infty)$, $\omega \in \Omega$.
Our main result is a following.

Theorem 1 *Suppose that there are $c_0, c_1 > 0$ such that the convex hull of $\{z_0, z_1, ..., z_N\}$ contains $\{z \in \mathbf{R}^N;\ | z | \leq 3c_1\}$ and that*

$$c_0|\xi| \leq \varphi(\xi, x) \leq c_1|\xi|, \qquad \xi \in \mathbf{R}^N,\ x \in \mathbf{R}^{1+N}.$$

Then there is a set $\mathcal{M}$ of probability measures on $C([0,\infty);\mathbf{R}^{1+N})$ independent of the function F such that

$$\pi_{h,[h^{-1}T]}(\{F(hk,W_h(\cdot))\}_{k=0}^{[h^{-1}T]})$$
$$\to \sup\{E^{\mu}[e^{-r\tau}F(\tau,w);\ \mu\in\mathcal{M},\ \tau \text{ is a stopping time with } \tau\le T\}$$

as $h\downarrow 0$.

To discribe the set $\mathcal{M}$, we need the notion of supermartingale problem.
Let $\mathcal{S}^d$ be a set of all symetric $d\times d$ matrices. Let $G:[0,\infty)\times\mathbf{R}^d\times\mathcal{S}^d\times\mathbf{R}^d\to\mathbf{R}^d$ be a continuous function such that $G(t,x,\cdot,*):\mathcal{S}^d\times\mathbf{R}^d$ is a convex function for any $(t,x)\in[0,\infty)\times\mathbf{R}^d$ and $G(t,x,A,0)\le 0$ for any nonpositive definite $d\times d$ symmetric matrix A and $(t,x)\in[0,\infty)\times\mathbf{R}^d$. We define a nonlinear operator L_G on $C_0^\infty(\mathbf{R}^d)$ by

$$L_Gf(t,x)=G(t,x,\{\frac{\partial^2 f}{\partial x_i\partial x_j}(x)\}_{i,j=1,\dots,d},\{\frac{\partial f}{\partial x_i}\}_{i=1,\dots,d}),$$

$$(t,x)\in[0,\infty)\times\mathbf{R}^d,\ f\in C_0^\infty(\mathbf{R}^d).$$

Definition 1 *We say that a probability measure ν in $C([0,\infty);\mathbf{R}^d)$ solves a super martingale problem associated with a generator L_G, (or associated with a function G) if $w(t)-\int_0^t L_Gf(w(s))\,ds$ $t\ge 0$ is a supermartingale under ν for any $f\in C_0^\infty(\mathbf{R}^d)$.*

Remark Let $a_{i,j}:[0,\infty)\times\mathbf{R}^d\to\mathbf{R}$, $i,j=1,\dots,d$ and $b_i:[0,\infty)\times\mathbf{R}^d\to\mathbf{R}$, $i=1,\dots,d$ be a contious function such that $\{a_{ij}(t,x)\}_{i,j=1\dots,d}$ is a nonnegative definite symmetric matrix for all $(t,x)\in[0,\infty)\times\mathbf{R}^d$, and define $G:[0,\infty)\times\mathbf{R}^d\times\mathcal{S}^d\times\mathbf{R}^d\to\mathbf{R}^d$ by

$$G(t,x,A,v)=\frac{1}{2}\sum_{i,j=1\dots,d}^{d}A_{ij}a_{ij}(t,x)+\sum_{i=1}^{d}v_ib_i(t,x).$$

Then the related supermartingale problem is a usual martingale problem.
Then the set $\mathcal{M}$ in Theorem is given by the set of solutions ν of a supermartiongale problem associated with $G:[0,\infty)\times\mathbf{R}^{1+N}\times\mathcal{S}^{1+N}\times\mathbf{R}^{1+N}$ with a initial condition that $\nu(w(0)=S_0)$. The function G is determined by a certain eigenvalue problem of a certain nonlinear operator and it is not easy to give a explicit expression.
Example Let $N=1$, $z_0=-1$, $z_1=1$, $\sigma>0$, $\alpha(x)=\sigma x$, $x\in\mathbf{R}$. Let $\kappa:\mathbf{R}^2\to(0,\infty)$ be a Lipschits continuous function and $\varphi(\xi,x)=\kappa(x)|\xi|$, $\xi\in\mathbf{R}$, $x\in\mathbf{R}^2$. Then the assumption of Theorem is satisfied if $\inf_{x\in\mathbf{R}^2}\kappa(x)>0$ and if $\sigma>3\kappa(x)$, $x\in\mathbf{R}^2$. In this case,

$$G(A,v,t,x)=\max\{A_{11}s+r(v_0+v_1);\ s\in[\sigma(\sigma-\kappa(x)),\sigma(\sigma+\kappa(x))]\}$$

and $\mathcal{M}$ is the set of solutions ν of supermartingale problem associated with the function G with the initial condition that $\nu(w(0)=S_0)=1$. If we think of American call option of the maturity T and the exercise price is c, then the functional F is given by $F(t,w)=(w^1(t)-c)_+$. This F satisfies our assumptions.
References

Boyle, P., and Vorst, T., Option replicationin discrete time with transaction costs, J. Finance 47(1992), 271-293
Cvitanic, J., and Karatzas, I., Hedging and portfolio optimization under transaction costs: a martingale approach, Mathematical Finance 6(1996), 133-165
Duffie, D., Dynamic Asset Pricing Theory, 2ed., Princeton UNiversity Press, 1996
Kusuoka, S., Limit theorem on option replication with transaction costs, Annals of Applied Probability 5(1995),198-221
Leland, H., Option pricing and replication with transaction costs, J. Finance 40(1985), 1283-1301

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The Forward Inequality and Applications

The forward inequality is a generalization of Kolmogorov's forward equation. This generalization aims at making Lyapunov's second method available in the study of infinite-dimensional diffusions with irregular coefficients. The Lyapunov functionals which we consider are lower semi- continuous and, as realvalued functions, only densely defined on the state space, as, e. g. quadratic forms induced by unbounded operators. The method can be applied to various questions on the qualitative behaviour of infinite-dimensional diffusions, as, e. g. non-explosion, existence and uniqueness of an equilibrium, attractivity, Feller property, and stability.

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On the Selection Properties of Set-valued Semimartingales

The stochastic optimal control theory, dealing with systems described by integral stochastic equations depending on random control parameters, is naturally connected with the theory of stochastic inclusions. This approach needs some special type of set-valued integrals generalizing a stochastic integral. The theory of set-valued stochastic integral has been studied by Hiai [1], Kisielewicz [3], [4], Kisielewicz and Sosulski [5] and Motyl [7]. These works consider properties of the integral $\int F_s dMs$, where F denotes a set-valued predictable process taking values in the space $Cl(\mathbb{R}^n)$ of nonempty, closed subsets of $\mathbb{R}^n$ and M is a one dimensional Brownian motion, martingale, martingale measure or semimartingale. One can consider the case where the integrator M is a set-valued process. To define such integral the set-valued process M should admit an appropriate selection, being at least a semimartingale. The aim of the presentation are set-valued semimartingales introduced as an extension of the notion of single-valued semimartingales. For such multivalued processes we define stochastic integral and then we investigate their selection properties.

Literatur

[1] F. Hiai, *Multivalued stochastic integrals and stochastic differential inclusions*, Division of Applied Mathematics, Research Institute of Applied Electricity, Sapporo 060, Japan, (preprint).

[2] F. Hiai, H. Umegaki, *Integrals,conditional expectations,and martingales of multivalued functions*, J. Multivar. Anal. **7** (1977), 149–182.

[3] M. Kisielewicz, *Properties of solution set of stochastic inclusion*, J. Appl. Math. Stoch. Anal. III, **6** (1993).

[4] M. Kisielewicz, *Set-valued stochastic integrals and stochastic inclusions*, Stoch. Anal. Appl. 16, 5 (1997) 783–800.

[5] M. Kisielewicz, W. Sosulski, *Set-valued stochastic integrals over martingale measures and stochastic inclusions*, Discuss. Math., **15** (2) (1995), 179–188.

[6] M. Michta, L. Rybiński, *Selections of set-valued stochastic processes*, J. Appl. Math. Stoch. Anal. 11, 1 (1998) (in press).

[7] J. Motyl, *Note on strong solutions of a stochastic inclusion*, J. Appl. Math. Stoch. Anal. III **8** (1995), 291–297.

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The minimal entropy martingale measure and the family of q-optimal martingale measures

We study the family of q-optimal martingale measures for a continuous semimartingale X. The densities of these measures minimize the L^q-norm of all signed martingale measures for X. In case that X is a diffusion process with stochastic volatility we prove that the densities of the q-optimal measures converge for $q \to 1$ in $L^1(P)$ to the density of the minimal entropy martingale measure which minimizes the Kullback-Leibler information of all equivalent martingale measures relative to the original measure. Futhermore we give an interpretation of this result in the context of hedging strategies in incomplete markets.

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How Imperfect Information Leads to Complete Asset Markets

Imperfect Information leads generically to a complete asset market in equilibrium if the rate of dividend is a diffusion process.
The framework for this result is as follows: a stochastic economy is set up where the uncertainty is described by a set of factors $(X, Y^1, \ldots, Y^n)$ evolving according to a stochastic differential equation. Two assets are traded: the stock, paying dividends at a rate X and a (locally riskless) money market account. Since there are more factors than risky assets, the market would be incomplete in general in the sense that there are risks that cannot be hedged by the agents. If, however, the system (X, Y) is only partially observable, if only X is observed, the agents estimate Y on the basis of their observations. All risks agents want to hedge are now described in terms of the observed data X. Using results of filtering theory and a martingale representation theorem, we show that, in this setting, there exists an equilibrium with a complete asset market.

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Diffusions with singular drifts: a survey on some recent results

In the talk some recent results on singular diffusions will be presented. In particular, strong and Markov uniqueness problems in L^2 for Dirichlet operators on rigged Hilbert spaces will be discussed as well as the extension of the problem to the L^p–setting. As a direct application essential self–adjointness and strong uniqueness in L^p is obtained for the generator (with initial domain the bounded smooth cylinder functions) of the stochastic quantization process for Euclidean quantum field theory in finite volume $\Lambda \subset R^2$.

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Existence and uniqueness of (nonsymmetric) Dirichlet operators on L^1

Let L be a nonsymmetric operator of type $Lu = \sum a_{ij}\partial_i\partial_j u + \sum b_i \partial_i u$ on an arbitrary subset $U \subset R^d$. We analyse L as an operator on $L^1(U,\mu)$ where μ is an invariant measure, i.e., a possibly infinite measure μ satisfying the equation $L^*\mu = 0$ (in the weak sense). We explicitely construct, under mild regularity assumptions, extensions of L generating sub–Markovian C_0–semigroups on $L^1(U,\mu)$ as well as associated diffusion processes. We give sufficient conditions on the coefficients so that there exists only one extension of L generating a C_0–semigroup and apply the results to prove uniqueness of the invariant measure μ. Our results imply in particular that if $\varphi \in H^{1,2}_{loc}(R^d, dx)$, $\varphi \neq 0$ dx–a.e., the generalized Schrödinger operator $(\Delta + 2\varphi^{-1}\nabla\varphi \cdot \nabla, C_0^\infty(R^d))$ has exactly one extension generating a C_0–semigroup if and only if the Friedrich's extension is conservative. We also give existence and uniqueness results for a corresponding class of infinite dimensional operators acting on smooth cylinder functions on a separable real Banach space.

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Euclidean Gibbs States for Quantum Lattice Systems and Assosiated Stochastic Dynamics

Models of quantum mechanical anharmonic lattice systems ("anharmonic crystals") are considered. Temperature quantum Gibbs states are represented by classical Gibbs measures for lattice systems of loop- valued spin variables. These Gibbs measures are also obtained as invariant (equilibrium) measures of a system of stochastic differential equations ("stochastic dynamics"). Existence and uniqueness results for these equations are established. The properties of the Gibbs distributions (existence, a-priori estimates, regularity of support) are characterized in terms of the stochastic dynamics. Ergodicity and uniqueness of the Gibbs distributions are also discussed.

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Quadraturformeln für das Wienermaß

Zur näherungsweisen Berechnung von Erwartungswerten bezüglich des Wienermaßes benützt man meist Monte-Carlo-Verfahren. Diese Verfahren sind ziemlich zuverläßig, aber langsam, selbst dann, wenn der Integrand sehr glatt ist.
In letzter Zeit habe ich mit K. Ritter (Erlangen) interpolatorische Methoden zur Berechnung hochaber endlichdimensionaler Integrale entwickelt. Die theoretische Analyse zeigt ebenso wie numerische Beispiele, daß im Fall glatter Integranden die neue Methode sehr vorteilhaft ist. Sie wird jetzt so weiterentwickelt, daß sie für Funktionalintegrale bezüglich des Wienermaßes anwendbar ist.
Bei gewissen Integralen aus dem Finanzbereich (Bewertung von Derivaten) erweist sich die neue Methode als sehr vorteilhaft.

Gemeinsame Arbeit mit Klaus Ritter und Achim Steinbauer.