Fraktale und Stochastik

Sektionsleiter: S. Graf, Passau

Bandt , Christoph
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A Simple Construction of Tangential Measure Distributions for Self-similar Measures

Bluhm , Christian
Cornell University

Fourier Asymptotics of Statistically Self-similar Measures

Keller, Gerhard
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Ein neuer Schätzer für die Informationsdimension – asymptotische Normalität und Konfidenzintervalle

Ein neuer Kleinste-Quadrate-Zugang zur Schätzung der Informationsdimension invarianter Maše chaotischer dynamischer Systeme wird vorgestellt. Er ähnelt dem Grassberger-Procaccia Algorithmus zur Schätzung der Korrelationsdimension über einem festen Bereich von Radien. Unter Mischungsannahmen an die Beobachtungen, die für chaotische dynamische Systeme üblich sind, hat dieser Schätzer die gleichen asymptotischen Normalitäts- eigenschaften wie das Grassberger-Procaccia Verfahren. Die Praktikabilität des Verfahrens wird durch Simulationsstudien belegt. Auf beweistechnischer Ebene hat man es mit einem „Zwitter“ aus U- und L-Statistiken zu tun, mit dessen Hilfe getrimmte räumliche Korrelationsintegrale geschätzt werden, die wiederum die Basis für ein KQ-Verfahren bilden.

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Stochastics over p-Adic Structures

The field of p-adic numbers and related structures (its ring of integers, group of units, finite and infinite extensions etc.) constitute a class of fractal objects admitting rich harmonic analysis which can be employed in the construction and study of stochastic objects with potential applications in mathematical physics and information technologies.
The talk will include a brief review of p-adic stochastics – stochastic differential equations and Markov processes over p-adics, limit theorems for sums of p-adic random variables etc.

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Asymptotischer Quantisierungsfehler für Wahrscheinlichkeitsverteilungen

Für einen $\mathbb{R}^d$-wertigen Zufallsvektor X mit Verteilung P ist

$$V_{n,r}(P) = \inf E\|X - f(X)\|^r$$

der n-te Quantisierungsfehler der Ordnung r, wobei das Infimum über alle meßbaren Abbildungen $f : \mathbb{R}^d \to \mathbb{R}^d$ mit $|f(\mathbb{R}^d)| \leq n$ gebildet wird. Der Quantisierungsfehler konvergiert gegen $0, \quad n \to \infty$. Die korrekte Konvergenzordnung ist $n^{-r/D}$ für diverse Klassen von Verteilungen ($D =$ Ähnlichkeitsdimension), Hausdorffmaße auf rektifizierbaren Kurven. Während für nicht-singuläre Verteilungen P der Grenzwert

$$Q_r(P) = \lim_{n\to\infty} n^{-r/D} V_{n,r}(P)$$

existiert, gilt dies z.B. für selbstähnliche Verteilungen i.a. nicht. Die Berechnung des r-ten Quantisierungskoeffizienten $Q_r(P)$ hängt eng mit geometrischen Problemen für raumfüllende Figuren zusammen. Das geometrische Überdeckungsproblem durch Kugeln mit gleichem Radius ergibt sich als Grenzfall des Quantisierungsproblems.

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Homogenization on Fractals

The construction of many fractals can be based on recursive self-similar cutouts from a homogeneous material. For example the so called Vicsek set X: Start with the unit square $S := [0,1]^2$, divide it into 9 smaller squares and cut out the open middle square, repeat the procedure with the remaining smaller squares recursively to get a sequence (X_n), the limit set $\cap_{n=0}^{\infty} X_n$ is the fractal X.
This can be viewed as polluting a homogeneous material of conductivity say 1 by inclusions of another material of conductivity 0. Let us call the four corner points of S the boundary B of X. Now the idea of homogenization is: Since the local structure of X is complicated, we only consider its global properties, i.e. applying voltages to B we are only interested in the amount of current flowing in or out at points of B. Very often these "black box data" of X can be viewed as coming from another homogenuous material with conductivity different from 1 or 0.
We will use potential theory in the form of Dirichlet forms and Markov processes to make this idea mathematically precise. Existence and uniqueness results for the limiting model as well as convergence results for the sequence of recursive approximations to X will be proved. The arguments that are going to be used are actually valid for a class of fractals called nested.

Mörters , Peter
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Average densities and tangent measure distributions of the path of a planar Brownian motion

Let μ be the occupation measure on the path of a planar Brownian motion run for an arbitrary finite time. I show that μ has an average density of order three with respect to the gauge function $\varphi(t) = t^2 \log(1/t)$, *i.e.* almost surely,

$$\lim_{\varepsilon \downarrow 0} \frac{1}{\log\log(1/\varepsilon)} \int_{\varepsilon}^{1/e} \frac{\mu(B(x,t))}{\varphi(t)} \frac{dt}{t\,\log(1/t)} = 2 \quad \text{at } \mu\text{-almost every } x.$$

The average density of order two fails to exist. More generally, I determine the tangent measure distributions of order three of μ: Denoting by $\mu_{x,t}$ the measure given by $\mu_{x,t}(A) = \mu(x+tA)$ and by ℓ^2 the Lebesgue measure, we have, almost surely, at μ-almost every x,

$$\lim_{\varepsilon\downarrow 0} \frac{1}{\log\log(1/\varepsilon)} \int_{\varepsilon}^{1/e} \delta_{\left\{\frac{\mu_{x,t}}{\varphi(t)}\right\}} \frac{dt}{t\,\log(1/t)} = \int_0^{\infty} \delta_{\{a\cdot\ell^2\}}\, a\, e^{-a}\, da\,,$$

where the convergence is weak convergence of distributions on the space of Radon measures with the vague topology. In other words, for almost every Brownian path, the distribution of the measure $\mu_{x,t}/\varphi(t)$ with respect to a random scale with logaritmic distribution of order three converges to the distribution of Lebesgue measure multiplied with a random constant, which is gamma distributed with parameter two. This result can be interpreted as a pathwise Kallianpur-Robbins law.

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Die strenge Offene-Mengen-Bedingung für zufällig sebstänliche Maße

Bei der Untersuchung fraktaler Eigenschaften von selbstähnlichen Mengen und Maßen wie Dimensionen oder Spektren ist es oft nützlich, daß neben der Offene-Mengen-Bedingung auch die sogenannte strenge Offene-Mengen-Bedingung erfüllt ist. Eine zufällige Menge Ξ heißt selbstähnlich, falls es zufällige Ähnlichkeitsabbildungen $S_1, \cdots, S_N$ gibt, so daß

$$\Xi =^d \bigcup_{i=1}^{N} S_i \Xi^i$$

gilt, wobei die Ξ^i unabh ngige zufällige Mengen mit der selben Verteilung wie Ξ sind. Analog heißt ein zufälliges Maß Φ auf Ξ selbstähnlich, falls es zufällige Zahlen $p_1, \cdots, p_N$ mit $0 < p_i < 1$ und $E\sum_{i=1}^{N} p_i = 1$ gibt, so daß

$$\Phi =^d \sum_{i=1}^{N} p_i\ \Phi^i \circ S_i^{-1}$$

gilt, wobei die Φ^i unabhängige Exemplare von Φ sind. Die Offene-Mengen-Bedingung ist erfllt, falls es eine offene Menge O gibt mit $S_i(O) \subset O$ und $S_i(O) \cap S_j(O) = \emptyset$ für $i \neq j$. Die strenge Offene-Mengen-Bedingung ist erfüllt, wenn außerdem $\Xi \cap O \neq \emptyset$ mit Wahrscheinlichkeit 1.
Im nichtzufälligen Fall ist bekannt, daß die Offene-Mengen-Bedingung und die strengen Offene-Mengen-Bedingung äquivalent sind. Wir zeigen hier, daß dies auch im zufälligen Fall gilt.

Taylor, S. James **Hauptvortrag**
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The fractal structure of stable measures

A stable subordinator of index α is an increasing process X_t with stationary independent increments such that

$$E\{\exp -u(X_{s+t} - X_s)\} = \exp(-tu^{\alpha})\ ,\ 0 < \alpha < 1.$$

This generates a stable measure on $\mathbf{R}$ - a random Borel measure given by

$$\mu(A) = |\{t \in [0,1] : X_t \in A\}|\,.$$

The topological support S of μ is the closed trajectory $\overline{X[0,1]}$. The precise density properties of μ at a typical point of S have been used to obtain the Hausdorff measure and the packing measure of the random set S. These results imply that, if we define the local dimension index at x to be

$$\begin{aligned} d(\mu, x) &= \lim \frac{\log \mu B(x,r)}{\log r}; \text{ and} \\ A_\beta &= \{x \in S : d(\mu, x) = \beta\}, \end{aligned}$$

then $\mu(A_\alpha) = \mu(S)$ with probability 1. We show that A_β is empty a.s. for $\beta \neq \alpha$ but there are interesting geometric properties of S which make C_β not empty for $\alpha \leq \beta \leq 2\alpha$, where

$$C_\beta = \{x \in S : \limsup \frac{\log \mu B(x,r)}{\log r} = \beta\};$$

while, in fact,

$$\liminf \frac{\log \mu B(x,r)}{\log r} = \alpha, \text{ for all } x \in S.$$

We have information about the Hausdorff and packing dimensions of C_β which are distinct for $\alpha < \beta < 2\alpha$.
To obtain interesting results about the points of S where μ is unusually dense, we consider

$$E_\theta = \left\{ x \in S : \limsup \frac{\mu B(x,r)}{c_\alpha r^\alpha (\log 1/r)^{1-\alpha}} = \theta \right\}$$

for an appropriate constant c_α. We show that E_θ is not empty for $0 \leq \theta \leq 1$, and

$$\dim E_\theta = \operatorname{Dim} E_\theta = \alpha(1 - \theta^{1/(1-\alpha)}) \text{ a.s.}$$

(Results on C_β were obtained jointly with X. Hu; those on E_θ with N-R. Shieh).

Zähle , Martina
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The average density of self-conformal measures

We present a formula calculating the average density of a self-conformal measure generated by a dynamical system in n-dimensional euclidean space. A computational estimate is derived.

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On Fractal Option Pricing

Finance theory is one of the most active fields in applied stochastics now. Especially the valuation of options in practice and theory has many surprises. Since the work of Black and Scholes many of those were attacked with much efforts.
In this talk we will focus our attention on the fractal aspects of the pricing of options manifesting itself in the self-similar structure of asset-return processes of real word stocks. Contrary to the geometric Brownian modell assumed in the classical theory of Black-Scholes there is much empirical evidence of fractal geometry in those time series. This ideas was postulated already by B.B. Mandelbrot in the sixties witin the framework of stable-laws.
We take up this idea in a more general setting and apply it to the pricing of European call options, where not only variances (volatilities) of the underlying stock are taken into account but also other natural parameters like fractal Hurst exponents.
As stochastic tool to achieve fair pricing we use the technique of exponential tilting (Esscher transform). The underlying stock model is within the very general framework of semi-stable stochastic processes.
This is joint work with Ralf Hendrych and Carl Evertsz both CeVis - University of Bremen.