Exercise Sheet 12, Advanced Algebra, Summer Semester 2017. To be discussed on Thursday 11.7.17

(Ehud Meir and Christoph Schweigert)

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1. Let $m,n,k,r \in \mathbb{Z}$. Assume that $n|rm$, so that $\phi : \mathbb{Z}/m \to \mathbb{Z}/n\; x \mapsto xr$ is a well defined homomorphism of abelian groups.

   (a) Calculate explicitly a lifting of ϕ from a projective resolution of $\mathbb{Z}/m$ to a projective resolution of $\mathbb{Z}/n$.

   (b) Calculate explicitly the maps $\phi_* : Tor_n^R(\mathbb{Z}/m,\mathbb{Z}/k) \to Tor_n^R(\mathbb{Z}/n,\mathbb{Z}/k)$ and $\phi^* : Ext_R^n(\mathbb{Z}/n,\mathbb{Z}/k) \to Ext_n^R(\mathbb{Z}/m,\mathbb{Z}/k)$ for $n = 0,1$.

2. Let $\mathcal{C}$ be an abelian category.

   (a) Show that the complex $\cdots 0 \to X \stackrel{Id}{\to} X \to 0 \to \cdots$ is a projective object in the category $Ch(\mathcal{C})$ if and only if X is projective in $\mathcal{C}$.

   (b) Let $C. = \cdots 0 \to X_n \to X_{n-1} \to \cdots \to X_0 \to 0 \to \cdots$ be a chain complex in some abelian category $\mathcal{C}$. Assume that C is projective. Show that C can be written as the direct sum of complexes from the first part of the exercise.

3. Bring your own questions, about everything that we have had during the semester!