Modal Realism and the Absolute Infinite

Christopher Menzel

Department of Philosophy
Texas A&M University
cmenzel@tamu.edu

History and Philosophy of Infinity Conference
Cambridge University
20-23 September 2013

Z Set Theory + Urelements (ZU)

Set $y \in x \rightarrow Set(x)$

- We write $\exists A\varphi^x_A$ for $\exists x(Set(x) \wedge \varphi)$ and $\forall A\varphi^x_A$ for $\forall x(Set(x) \rightarrow \varphi)$

Ext $\forall x(x \in A \leftrightarrow x \in B) \rightarrow A = B$

Pg $\exists A \forall z(z \in A \leftrightarrow z = x \vee z = y)$

Un $\exists C \forall x(x \in C \leftrightarrow x \in A \vee x \in B)$

Sep $\exists B \forall x(x \in B \leftrightarrow x \in A \wedge \varphi)$, '$B$' does not occur free in φ.

- We will avail ourselves of set abstracts $\{x : \varphi\}$ when we can prove $\exists A \forall x(x \in A \leftrightarrow \varphi)$.

Fnd $A \neq \emptyset \rightarrow \exists x \in A\, x \cap A = \emptyset$

Inf $\exists A(\emptyset \in A \wedge \forall x(x \in A \rightarrow x \cup \{x\} \in A))$

PS $\exists B \forall x(x \in B \leftrightarrow x \subseteq A)$

- Let $\wp(A) =_{df} \{x : x \subseteq A\}$.

ZFCU: ZU + Replacement and Choice

- Critical to set theory's power — and to matters here — is Fraenkel's axiom schema of Replacement:

F $\forall x \in A \exists! y\, \psi \rightarrow \exists B \forall y (y \in B \leftrightarrow \exists x (x \in A \wedge \psi))$, where '$B$' does not occur free in ψ

- The axiom of Choice simplifies matters considerably.
  - And it's true anyway!
- Say that x is *choice-friendly*, $CF(x)$, if x is a set of nonempty pairwise disjoint sets:

AC $CF(x) \rightarrow \exists C \forall B \in x \exists! z \in B\, z \in C$.

- Let ZFU = ZU + **F** and ZFCU = ZFU + **AC**

Ordinals and Size

- $Tran(x) \equiv_{df} Set(x) \wedge \forall A(A \in x \rightarrow A \subseteq x)$
- $PTran(x) \equiv_{df} Tran(x) \wedge \forall y(y \in x \rightarrow Set(y))$
- $Ord(x) \equiv_{df} PTran(x) \wedge \forall yz \in x(y \in z \vee z \in y \vee y = z)$
- $x < y \equiv_{df} Ord(x) \wedge Ord(y) \wedge x \in y$

Let α, β , and γ range over ordinals.

- $A \approx B \equiv_{df} \exists f\ f : A \xrightarrow[onto]{1\text{-}1} B$ (A *is as large as* B)
- $A \prec B \equiv_{df} \exists C(C \subseteq B \wedge A \not\approx B)$ (A is *smaller than* B)

- Given both **F** and **AC**, every set is the size of some ordinal:

Theorem (OrdSize): $\forall A \exists \alpha\, A \approx \alpha$

Cardinals and Cantor's Theorem

- $Card(x) \equiv_{df} Ord(x) \wedge \forall y(y < x \rightarrow y \prec x)$

Let κ and ν range over cardinals.

- By **OrdSize** and the w.o.-ness of the ordinals every set has a definite cardinality:
- $|A| = (the\ \kappa)\kappa \approx A$ (alternatively: $|A| = \{\alpha : \alpha \prec A\}$)
  - Absent **OrdSize**, we can take the cardinality operator to be a *façon de parler* : $\varphi(|A|) \equiv_{df} \exists\kappa\,(\kappa \approx A \wedge \varphi(\kappa))$

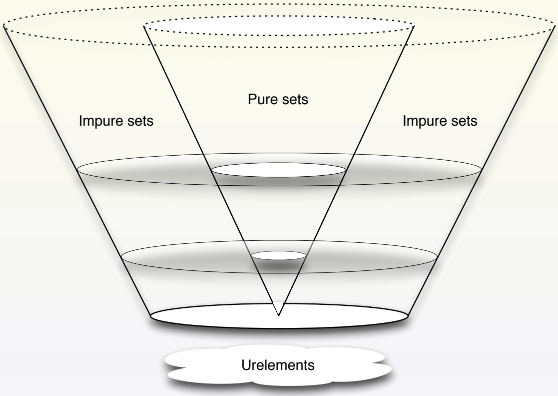
Theorem (Cantor): $\forall A\, A \prec \wp(A)$

- **Corollary:** $\forall A\, |A| < |\wp(A)|$

- It follows immediately that no set's cardinality is maximal:

Theorem (NoMax): $\forall A \exists \kappa\, |A| < \kappa$

The World According to ZFCU



How Many Atoms are There?

- Nolan [1] has shown that Lewis's [2] (unqualified) principle of Recombination commits him to more atoms than can be measured by any cardinal:

A$_\infty$ $\forall\kappa\exists A(\forall x(x \in A \rightarrow {\sim}Set(x)) \wedge \kappa \leq |A|)$

- Let **SoA** be the proposition that there is a set of atoms:

SoA $\exists A\forall x(x \in A \leftrightarrow {\sim}Set(x))$

- Let **SoA$_\infty$** be the conjunction **SoA** $\wedge$ **A$_\infty$**
- By **NoMax**, ZFCU $\vdash$ **~SoA$_\infty$**
  - In fact, ZU $\vdash$ **~SoA$_\infty$** via Hartogs' theorem if we replace "$\kappa \leq |A|$" with "$\kappa \preccurlyeq A$" in $\mathbf{A}_\infty$.
- But the inconsistency of **SoA$_\infty$** with ZFCU is a bit puzzling...

The Iterative Conception of Set

- The conception of set underlying ZFCU is the so-called *iterative* conception.
- Sets are "formed" in "stages" from an initial stock of atoms.
  - Stage 1: All sets of atoms are formed.
  - Stage $\alpha > 1$: All sets that can be formed from atoms and sets formed at earlier stages.
  - To be a set is to be formed at some stage.
- Less metaphorically:
  - The *rank* of an atom is 0.
  - Objects such that some ordinal α is the (strict) supremum of their ranks form a set of rank α.
  - To be a set is to have a rank.

Size vs Structure

- The crucial observation:

> Iterative sethood is not about *size* but about *structure*

- Objects constitute a set if and only if there is an upper bound to their ranks.
- Hence, since atoms have a rank of 0, no matter how many there are, there should be a set of them, i.e., **SoA** is true.
- The iterative conception only rules out collections that are "too high", i.e., unbounded in rank.
- Nothing in the conception that entails sets can't be at least as "wide" as the universe is high...
- ...hence, sets that are *mathematically indeterminable*, i.e., sets that, *qua* sets, have a definite rank but which are too large to have a definite cardinality

A Disconnect; and Some Questions

- So we seem to have a disconnect
- A_∞ is, at the least, conceptually possible
- But suppose it is true. Then:
  - Given the iterative conception: **SoA**
  - Given ZFCU: ~**SoA**
- But the iterative conception provides the conceptual underpinnings for ZFCU.
- Which leads us to wonder:
  - What, exactly, is the source of the apparent disconnect?
  - Can we modify ZFCU to accommodate **SoA**$_\infty$ without abandoning the iterative conception?
  - What are the philosophical implications of these modifications, e.g., vis-á-vis modal realism?

Awkward Consequences for Lewis

- Assuming that every Lewisian world w contains a definite number κ_w of things, in ZFCU, **~A** entails:

~W *There is no set of all worlds.*

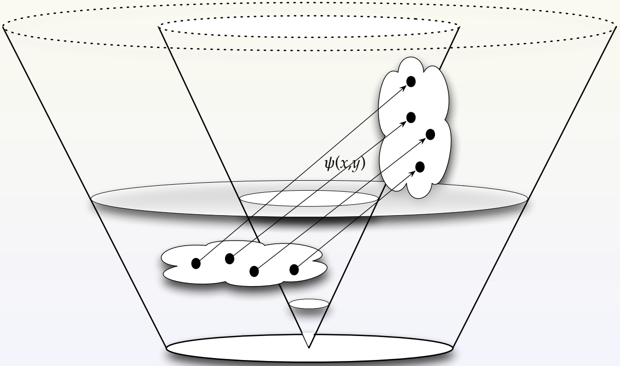
- Recall that for Lewis:
  - Properties are sets of concrete things
  - Propositions are sets of worlds
- Given **~SoA**, **~W**, and Recombination, many intuitive properties and propositions do not exist:
  - *being a concrete object*, *being a dog*
  - *that dogs exist*, the (one) necessary truth
- But Lewis accepts both the iterative conception and ZFCU and hence must modify Recombination to avoid **~SoA** and **~W**.
  - Justifies this with the (dubious?) claim that there is a bound on the number of objects that can "fit" into any possible spacetime (1986, 104)

The Central Culprit: The Replacement Schema **F**

- Boolos [3] and Potter [4] have noted that **F** is at best marginally warranted by the iterative conception
  - Their focus is on its power to generate ever higher levels of the iterative hierarchy.
  - The cause of the disconnect is the "flip side" of this capability.
- **F** guarantees that width and height grow in tandem.
- Otherwise put: ***F*** *is a double-edged sword:*
  1. Given a set S of any size, **F** *extends* the hierarchy by guaranteeing an upper bound to any way of mapping S "upward" (consider, e.g., $n \mapsto \aleph_n$).
  2. On the other hand, **F** *restricts* us to sets whose size does not outpace height (notably via **OrdSize**)
- **F** thus builds narrowness into the notion of set.

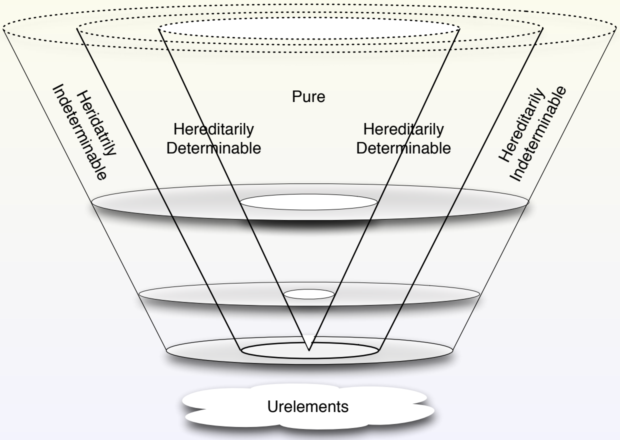
Replacement (F) and the World According to ZFU

- Under **F**, we cannot "replace" our way out of the universe under a functional operation ψ
- Hence, there can be no "wide" sets



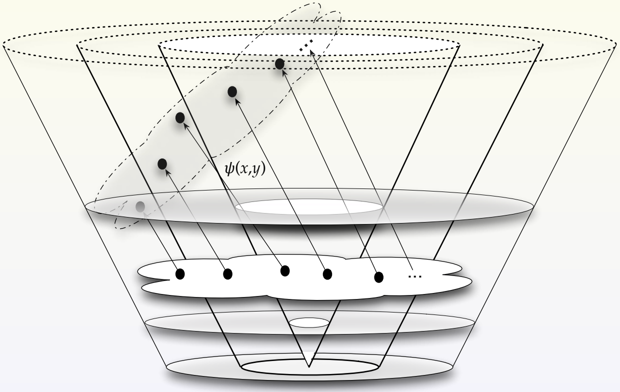
A World With "Wide" Sets

- So what would the world look like under the iterative conception under assumption A_∞?



Replacement (F) in a World with Wide Sets

- But for reasons just noted, the replacement schema **F** threatens to allow us to "replace" our way out of the universe on wide sets



- So **F** needs modification

Modifying F: Determinability

- Proposal: Restrict **F** so that only the ranges of operations ψ on determinable (i.e., "narrow") sets determine further sets:

F′ $Det(A) \rightarrow [\forall x \in A \exists! y\, \psi \rightarrow \exists B \forall y (y \in B \leftrightarrow \exists x \in A\, \psi)]$

- where we use the pure sets as "yardsticks" of determinability:

$$Det(x) \equiv_{df} \exists y (Pure(y) \wedge x \approx y)$$

- where a pure set is one that has only sets in its transitive closure.
  - $Pure(x) \equiv_{df} \exists y (TC(x,y) \wedge \forall z (x \in y \rightarrow Set(z)))$, where
  - $TC(x,y) \equiv_{df} x \subseteq y \wedge Tran(y) \wedge \forall z ((Tran(z) \wedge x \subseteq z) \rightarrow y \subseteq z)$ [1]

[1] TC is defined as a relation because recursion on ω requires **F′**. We can prove later that every set has a unique transitive closure.

Definition by Recursion and the Rank Function

- **F′** suffices for legitimizing definitions by recursion on the ordinals.
  - It is possible to prove $\forall\alpha Det(\alpha)$ without **F′**.[2]
- It does not suffice for general definitions by recursion on well-founded relations, which can involve wide sets, notably:

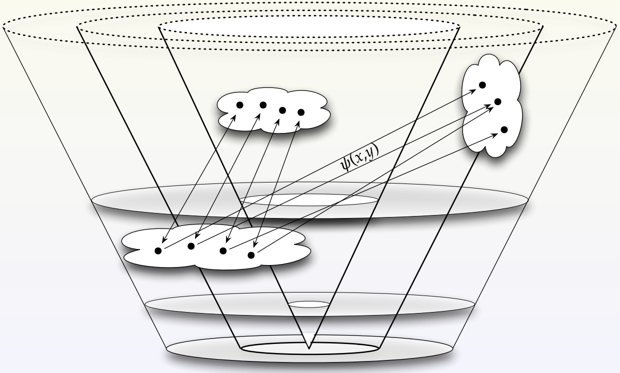
Rnk $rnk(x) = sup^{+}\{rnk(y) : y \in x\}$

- Solution: Fittingly, given its fundamental conceptual role in the iterative conception of set, Take **Rnk** as an axiom, with 'rnk' as a primitive symbol.
- Let ZFCU′ = ZU + **AC** + **Rnk** + **F′**

[2] Key is proving $\forall A \exists! B\, TC(A, B)$, which follows from **Inf**, **Fnd**, and **Sep**.

Replacement (F′) in the World according to ZFCU′

- ZFCU′ is obviously no stronger than ZFCU
- But the restriction on Replacement renders the proof of **OrdSize** unsound and, hence, renders ZFCU′ consistent with $\mathbf{SoA}_\infty$



A Model of $\mathrm{ZFCU}' + \mathbf{SoA}_\infty$

- Let ZFC^+ be ZFC + "There is an inaccessible cardinal".
- Let κ be the first inaccessible
- Let $A = \{\langle\kappa,\alpha\rangle : \alpha < \kappa\}$, where $\langle\kappa,\alpha\rangle = \{\{\kappa\},\{\kappa,\alpha\}\}$
- For $\alpha < \kappa$ and limit ordinals $\lambda \leq \kappa$, let:

$$\begin{aligned} A_0 &= A \\ A_{\alpha+1} &= A_\alpha \cup \wp(A_\alpha) \\ A_\lambda &= \bigcup_{\alpha<\lambda} A_\alpha \end{aligned}$$

- Let $\boldsymbol{A}' = \langle A_\kappa, \in\restriction A_\kappa\rangle$. '*Set*' in $\boldsymbol{A}'$ picks out $A_\kappa \setminus A$; '*Det*' picks out $\{B \in A_\kappa : |B| < \kappa\}$.
- Easy to see that A_∞, **SoA**, and all instances of F' are true in $\boldsymbol{A}'$
  - Let ψ be a functional mapping on a "determinable" set $B \in A_\kappa$ and let $C = \{y : \exists x \in B\, \psi(x,y)\} \subseteq A_\kappa$ be the range of ψ on B. $\{rnk(y) : y \in C)\} \subseteq \kappa$ is of cardinality $\leq |C| \leq |B| < \kappa$ and, hence, is not cofinal in (inaccessible) κ, so $\beta = sup^+\{rnk(y) : y \in C)\} < \kappa$. So $C \subseteq A_\beta$, and hence $C \in A_{\beta+1} \subseteq A_\kappa$.

Problems for ZFCU′: Number and Relative Size

- The ZFCU′ solution is in several ways unlovely.
- Most obviously because of the Powerset axiom **PS**.
- Suppose we assume a wide set A^* of urelements.
- Then by Cantor's theorem, $\wp(A^*)$ will be *strictly larger* in the sense that $A^* \prec \wp(A^*)$; likewise $\wp(A^*) \prec \wp\wp(A^*)$; etc
- But since **OrdSize** fails in ZFCU′, neither A^* nor $\wp(A^*)$ has a definite cardinality.
- But what else does a progression of propositions of the form $A \prec B, B \prec C, \ldots$ indicate than a progression of increasing *sizes*?
- And what else can such increases in size be but increases in *cardinality*?
- Hence, ZFCU′ yields an untenable picture of the set theoretic universe.

The Absolutely Infinite as a Quantitative Maximum

- Cantor himself recognized that some collections are indeterminable, or *absolutely infinite*
  - Notably, the collection On of all ordinals.
- Such collections represent an "absolute quantitative maximum" that is incapable of definite increase.
- This inspired *limitation of size* approaches to paradox.
- But these approaches are often ham-handed insofar as they conflate size and structure.
  - Cf. von Neumann's axiom that all and only proper classes (i.e., collections of unbounded rank) are the size of the universe).
- The iterative conception doesn't provide any justification for ruling out wide sets.
- But insofar as the universe grows "upward" in concert with the ordinals it it is entirely compatible with a notion of *size capable/incapable of increase*...

Powerset and the Absolutely Infinite

Inc *All and only determinable sets are of a size capable of definite increase.*

- That is, all and only such sets can be determinately smaller than another set.

- At a minimum, then, Powerset needs to be modified vis-á-vis wide sets to accommodate **Inc**
- It is needlessly strong to restrict it to determinable sets
- There is increase only if *all* subsets of a set, determinable and indeterminable alike, are taken to constitute a set; thus:

PS* $\forall A \exists B \forall x (x \in B \leftrightarrow Det(x) \wedge x \subseteq A)$

- Assuming $\forall A\, Det(A)$, **PS*** and **PS** are of course equivalent
- Without that assumption, Cantor's theorem fails in general

Ruling out Increase in the Absolutely Infinite

- With **PS*** there is no provable "expansion" of the hierarchy from stage to stage.
- But it is only *compatible with* a maximal absolute — it doesn't *express* it.
- One possibility:

Max $\neg Det(A) \rightarrow (\neg Det(B) \rightarrow A \approx B)$

- But this axiom, like von Neumann's, itself constitutes an unjustifiably definite fact about the absolutely infinite.
- A more modest proposal: Only determinable sets can be smaller than other sets:

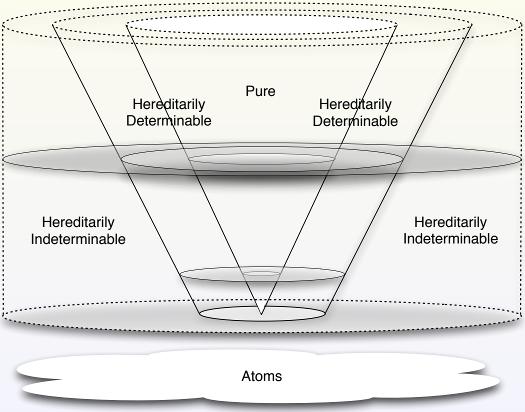
Det $A \prec B \rightarrow Det(A)$

Likewise Choice?

- Bottom line: There does not appear to be a parallel case for restricting **AC** to rule out w.o. sets too large to have an order type.
  - Suppose $\mathbf{SoA}_\infty$; by **AC** there is a well-ordering R of the set A^* of atoms.
  - Let a_0 the R-least element of A^*. Then, by **PS*** and **Sep**, define R^+ so that, for $x, y \in A^*$, $R^+(x,y)$ iff $x \neq a_0$ and either $R(x,y)$ or $y = a_0$.
  - $\langle A^*, R\rangle$ is therefore "shorter than" $\langle A^*, R^+\rangle$
  - There are, however, no corresponding well-order types, no corresponding ordinals that "measure" these orderings
  - But what else can these increases in "length" indicate than increases in *order type*?
  - There is a flaw in this reasoning: Proof the well-ordering theorem depends (essentially, I *believe*) on full **PS**.
    - A w.o.ing of A is constructed via a choice function on $\wp(A) \setminus \{\varnothing\}$.

The World with *Absolutely Infinite* Sets

Absolutely infinite sets exhibit a quantitative maximum incapable of mathematically definite increase.



New Limitations

- Our modified axioms **PS*** and **F′** leave us unable to prove the existence of certain intuitively unproblematic sets
- E.g., $\{A^* \setminus \{a\} : a \in A\}$, assuming A^* is wide.
  - Suppose the set A^* of all atoms is wide and, for $a \in A^*$, let $A^*_a = A^* \setminus \{a\}$.
  - Can't prove that the range $\{A^*_a : a \in A^*\}$ of the mapping $a \mapsto A^*_a$ exists.
    - **F′** useless because A^* is indeterminable.
    - Can't extract by **Sep** from $\wp^*(A^*)$ because its members are determinable and hence do not include the A^*_a.
- But all the A^*_a are "available" at stage 1 and there no more of them than atoms.
- So there is no principled objection to its existence.

Broadening Replacement: Boundedness

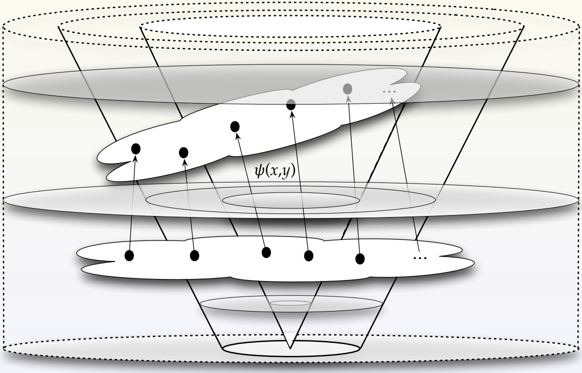
- Limiting Replacement to determinable sets is too strong.
- The purpose of the limitation was to heed the central structural constraint on set formation, viz., boundedness in rank.
- But this constraint is also satisfied if we can establish the boundedness of a mapping on A independent of A's size.
- Hence, say that ψ is *bounded above on* a set A, $BA(\psi, A)$, just in case there is an upper bound on the ranks of the objects in the range of the mapping:
  - $BA(\psi, A) =_{df} \exists\alpha\forall x \in A(\psi \rightarrow rnk(y) < \alpha)$.

F* $(Det(A) \vee BA(\psi, A)) \rightarrow [\forall x \in A\exists! y\, \psi \rightarrow \exists B\forall y(y \in B \leftrightarrow \exists x \in A\, \psi)]$

- Let ZFCU* be the result of replacing **F′** with **F*** and **PS** with **PS*** in ZFCU′.

Replacement (F*) in the World according to ZFCU*

The objects in the range of a functional mapping constitute a set if there is a bound on their ranks.



A Model of ZFCU* in ZFC$^+$

- Let ZFC$^+$, κ, and $A = \{\langle \kappa, \alpha \rangle : \alpha < \kappa\}$ be as before.
- For $\alpha < \kappa$ and limit ordinals $\lambda \leq \kappa$, let:

$$\begin{aligned} A_0 &= A \\ A_{\alpha+1} &= A_\alpha \cup \{B \subseteq A_\alpha : |B| \leq \kappa\} \\ A_\lambda &= \bigcup_{\alpha<\lambda} A_\alpha \end{aligned}$$

- Let $\boldsymbol{A}^* = \langle A_\kappa, \in \restriction A_\kappa \rangle$
- '*Set*' in $\boldsymbol{A}^*$ picks out $A_\kappa \setminus A$
- '*Det*' in $\boldsymbol{A}^*$ picks out $\{B \in A_\kappa : |B| < \kappa\}$
- '*rnk*' in $\boldsymbol{A}^*$ picks out the function $\rho : A_\kappa \longrightarrow \kappa$ such that $\rho(x) = sup^+\{\rho(y) : y \in \boldsymbol{A}^* \wedge y \in x\}$.
- **A$_\infty$**, **SoA**, **PS***, **AC***, **Rnk** and all instances of **F*** are true in $\boldsymbol{A}^*$.

An Objection

PS* *reintroduces, one level up, the same tensions with the iterative conception that motivated our project in the first place.*

A Reply

- Iterative conception tells us that each stage consists of all the sets that *can be* formed from the urelements and the sets from previous stages.
- Unrestricted Powerset (**PS**) is a way of making this idea concrete.
- But it begs the question to insist that **PS** is *constitutive* of the stage-by-stage growth of the iterative conception *from any starting point*.
- We really have no clear grasp of **PS;** witness Easton's theorem:
  - $|\wp(\mathbb{N})| = 2^{\aleph_0} = \aleph_\alpha$ is consistent with ZFC for *any* $\alpha > 0$ not of cofinality ω
- Hallett [5] (p. 208): Powerset "is just a mystery".

A Positive Argument for **PS***

- **PS** is obvious and, indeed, unnecessary for finite sets.
- The fact that $|\mathbb{R}| = |\wp(\mathbb{N})|$ provides a powerful justification for **PS** applied to countable sets.
- Likewise higher analysis provides grounds for extending into the uncountable and thence, arguably, to determinable sets generally.
- There would be little reason to accept **PS** for even for countable sets if it weren't for the fact that $|\mathbb{R}| = |\wp(\mathbb{N})|$.
- But there are simply no concrete examples of a powerset-related connection between indeterminable collections.
  - Certainly nothing to override the arguments for an absolute quantitative maximum.
- Hence: **PS*** is justified; burden is on **PS** to justify applicability to absolutely infinite sets.

Applications to Modal Realism I: Paradox

- Numerous paradoxes resulting from unrestricted Recombination rely upon the applicability of full Powerset to the set of worlds or to certain sets of individuals.
- The argument of Forrest and Armstrong (discussed in *PoW*, §2.2) explicitly involves a cardinality argument.
  - Specifically, that the number of electrons in the "Big World" that includes duplicates of every world is assumed to have a definite cardinality .
- Lewis's own version of the Kaplan paradox in *PoW* §2.3 also relies on full Powerset as well as the assumption that the set of worlds has a definite cardinality.
  - (Shameless self-promotion: Bueno, Menzel, and Zalta [6] show that the critical principle in Kaplan's paradox is a logical falsehood and, hence, that there is no genuine paradox.)

Applications to Modal Realism II: Propositions and Properties

- The intitial problem presented by Nolan's argument is that many intuitive properties and propositions needed to serve the semantic values of many ordinary language expressions as cannot exist simply in virtue of being absolutely infinite.
  - E.g., the proposition *that dogs exists* or the property *being a dog*.
- But under ZFCU, Lewis gets a set A of all individuals and a set W of all worlds
- Hence, we get back such propositions as *There are dogs* and *being a dog*.
  - $\{w \in W : \exists x \in w\ Dog(x)\}$, $\{a \in A : Dog(a)\}$.

Applications to Modal Realism III: Semantics

- Lewis doesn't get a set of all propositions ($= \wp(W)$) or a set of all properties of individuals ($= \wp(A)$) under ZFCU.
- Hence, we can't in general assign denotations to higher syntactic types.
  - E.g., the determiner *every* is $[\lambda F \lambda G\, \forall x(Fx \rightarrow Gx)]$.
  - We can't prove the existence of its usual denotation $\{\langle B, C\rangle \in \wp(A \times A) : B \subseteq C\}$
- However, we can still *quantify over* all of the the subsets of W, A as well as the "members" of other second-order "semantic types", even if they are never collected into a set.
- Thus, it is perhaps enough for most semantic purposes to give *every* a "syncategorematic" semantics and simply say it is *true of* those $\langle B, C\rangle$ such that $B \subseteq C$.
  - And that the quantifier *every dog* — $[\lambda G\, \forall x(Dog(x) \rightarrow Gx]$ — is true of those C such that $\{a \in A : Dog(a)\} \subseteq C$.
  - This account appears to suffice for the semantic applications of §1.4 in *PoW*.
- Obvious (critical?) limitation: Can't define higher types that take second-order types as arguments.
  - Hence can't replicate the framework of "General Semantics" as is.

(Tentative!) Applications to Absolute Generality

- “A concept is *indefinitely extensible* if, for any definite characterization of it, there is a natural extension of this characterisation which yields a more inclusive concept.” (Dummett [7])
- “[T]he concept of ‘set’ itself is also indefinitely extensible in this sense: given any (precisely specified?) totality of sets, that totality itself behaves intuitively like a set: it is identified by its members, and it can be subject to further set-theoretic operations, e.g. forming its singleton, taking its power set, etc.” (Hellman [8])
- Claim: the concept *concrete object* (the non-sets, in the context of modal realism) is not indefinitely extensible.
    - Sum formation is “closed” after one iteration
    - The sum of all objects is *already* an object

Applications to Absolute Generality

- But we can consistently assume (in 2nd-order ZFCU*):

There is a 1-1 mapping F from concrete things onto the sets.

$$\exists F(\forall x\, Set(Fx) \wedge \forall y(Set(y) \rightarrow \exists! x({\sim}Set(x) \wedge Fx = y)))$$

- The possibility of an absolutely infinite set of concrete things being mapped onto the sets seems to "anchor" the extension of the concept *Set*.
    - If *Set* were indefinitely extensible, it seems that it shouldn't even be possible consistently to postulate that that concept is in one-to-one correspondence with an essentially *non-extensible* concept.
    - At the least, *contra* some skeptical accounts, this seems to show that absolute generality is *coherent*.

Thanks...

- Hannes Leitgeb and the Munich Center for Mathematical Philosophy
- Humboldt Stiftung
- Organizers of this fine conference!

References I

[1] D. Nolan, "Recombination Unbound," *Philosophical Studies*, vol. 84, no. 2/3, pp. 239–262, 1996.

[2] D. Lewis, *On The Plurality of Worlds*. New York: Oxford University Press, 1986.

[3] G. Boolos, "Iteration Again," *Philosophical Topics*, vol. 17, no. 2, pp. 5–21, 1989.

[4] M. Potter, *Set Theory and Its Philosophy*. Oxford University Press, 2004.

[5] M. Hallett, *Cantorian Set Theory and Limitation of Size*. Oxford: Clarendon Press, 1984.

[6] O. Bueno, C. Menzel, and E. Zalta, "Worlds and Propositions Set Free."

References II

[7] M. Dummett, "The Philosophical Significance of Gödel's Theorem," in *Truth and Other Enigmas*. Cambridge, MA: Harvard University Press, 1978, pp. 186–201.

[8] G. Hellman, "Against 'Absolutely Everything'!" in *Absolute Generality*, A. Rayo and G. Uzquiano, Eds. Oxford: Oxford University Press, 2006, no. November, pp. 75–97.