WHEN SERIES GO IN INDEFINITUM, AD INFINITUM AND IN INFINITUM

CONCEPTS OF INFINITY IN KANT'S PHILOSOPHY AND COSMOLOGY

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CONTENTS

PART I

Kant’s Antinomy of pure Reason and Cosmology

THE «LOGIC» BEHIND THE ANTINOMY

- The concept of series (Reihe)
- Distinction between datum, dabilis, non datum non dabilis

«Let there be a series m, n, o, in which n is given as conditioned with respect to m, but at the same time as the condition of o, and the series ascends from the conditioned n to m (l, k, j, etc.); then I must suppose the first series in order to regard n as given, and n is possible in accordance with reason (with the totality of conditions) only by means of that series; but its possibility does not rest on the subsequent series o, p, q, r, which therefore cannot be regarded as given, but only as dabilis».

Kant, KrV A 410-11/B437-38.

THE «LOGIC» BEHIND THE ANTINOMY

- Two kinds of synthesis of the series: regressive (toward the more remote conditions) and progressive (from proximate consequences to the more remote ones)

- THE COSMOLOGICAL IDEAS ARE CONCERNED WITH THE TOTALITY OF THE REGRESSIVE SYNTHESIS

THE «LOGIC» BEHIND THE ANTINOMY

There are 4 cosmological ideas that necessarily carry with them a series in the synthesis of the manifold

- 1. the absolute completeness of the composition of a given whole of all appearances
- 2. the absolute completeness of the division of a given whole in appearance
- 3. the absolute completeness of the arising of an appearance in general
- 4. the absolute completeness of the dependence of the existence of the alterable in appearance

- These ideas give rise to dilemmas and to the «Antithetic of pure reason»

THE «LOGIC» BEHIND THE ANTINOMY

- The entire antinomy rests on the dialectical argument:

IF THE CONDITIONED IS GIVEN

THEN THE WHOLE SERIES OF ALL CONDITIONS FOR IT IS ALSO GIVEN

- The major premise of the cosmological syllogism takes the conditioned in the transcendental signification of a pure category, while the minor premise takes it in the empirical signification of a concept of the understanding applied to mere appearances (SOPHISMA FIGURAE DICTIONIS)

THE ANTINOMY OF PURE REASON

ABSOLUTE COMPLETENESS OF THE COMPOSITION OF A GIVEN WHOLE OF APPEARANCES

Thesis I

The world has a beginning in time, and in space it is also enclosed in boundaries

Antithesis I

The world has no beginning and no bounds in space, but is infinite with regard to both time and space

ABSOLUTE COMPLETENESS OF THE DIVISION OF A GIVEN WHOLE IN APPEARANCE

Thesis II

Every composite substance in the world consists of simple parts, and nothing exists anywhere except the simple or what is composed as simples

Antithesis II

No composite thing in the world consists of simple parts, and nowhere in it does there exist anything simple

KANT'S COSMOLOGY

- The universe is an expanding sphere (progressive synthesis)
- The expansion is indefinite in space and time (progression and regression)
- The origin of the universe can be known by assuming that matter set up into motion through the forces of attraction and repulsion: the cause of the universe as a whole cannot be known, how matter started moving can be an object of our knowledge or an indirect phenomenon (universality and necessity of the laws of physics)

A TROUBLE?

How do we reconcile Kant's cosmology with the antinomy?

«The merely general representation of the series of all past states of the world, as well as of the things that simoultaneously exist in the world's space, is nothing other than a possible empirical regress that I think, though still indeterminately, and through which alone there can arise the concept of such a series of conditions for a given perception.»

Kant, KrV A518/B546

PART II

Concepts of infinity in Kant’s antinomy

SERIES AND REGRESSUM: THREE CONCEPTS OF INFINITY

1. **Regressum in indefinitum** (philosophers): if only one member of the series is given, from which the regress to an absolute totality is to proceed, then only an indeterminate kind of regress takes place

Example: series of ancestors for a given human being. The series goes to an indeterminate distance, searching for more members for the given, which are once again always given only conditionally.

2. **Regressum in infinitum** (mathematicians): regress of decomposition (division of the given)

3. **Regressum ad infinitum**: regress of composition (composition of the given)

- Historical sources:

Descartes (1644), Principia Philosophiae, Ouvres VIII, pp. 14-15

Baumgarten (1757), Metaphysica, §248

PART III

Regressum and cosmological principle

KANT'S COSMOLOGICAL PRINCIPLE

- A given conditioned implies that A REGRESS IN THE SERIES OF ALL CONDITIONS IS GIVEN TO US AS A PROBLEM
- THE WORLD AS A WHOLE (as a thing in itself) IS NOT AN OBJECT OF POSSIBLE EXPERIENCE (IT CANNOT BE GIVEN IN THE INTUITION or be an object of direct measurement or a determined magnitude, therefore it cannot be neither finite nor infinite)
- Cosmological principle: regulative, rule prescribing a regress in the series of conditions for given appearances , in which regress it is never allowed to stop with an absolutely unconditioned (KrV A508-9/B536-37)
- **The only regress allowed is in indefinitum (Kant, KrV A 524/B552)**
- **What about the progress of a series?**

Conclusion and Remarks:

Reading Kant’s Critique of pure Reason through Zermelo’s eyes

FOLLOWING KANT'S FOOTPRINTS: ZERMELO ON THE ANTINOMY

"Human reason, by virtue of its inner nature, finds itself pressed to view the world simultaneously as limited and as unlimited, as finite and as infinite. And no mathematical theory will rid us of this fact" (Zermelo, 1932).

The sequence of ordinal numbers is apprehended simultaneously as limited (closed) and unlimited (open). Misunderstanding of this dialectic as source of the "ultrafinite paradoxes."

FOLLOWING KANT'S FOOTPRINTS: ZERMELO ON THE ANTINOMY

"The two opposing tendencies of the life of the mind—on the one hand, the concept of *creative progress* [schöpferischer Fortschritt] and, on the other, that of *inclusive closure* [zusammenfassender Abschluß]—are the roots of the Kantian antinomies; both tendencies find their symbolic representation as well as their symbolic resolution in the transfinite number sequence, which is grounded in the well-ordering concept. The transfinite number sequence, in its turn, exhibits no genuine closure in its unbounded advance but, rather, only interim stopping points, namely, those strongly inaccessible that mark off the higher models from the lower ones. And thus the set-theoretic antinomies, properly understood, lead not to any narrowing or truncation but, rather, to an unsurveyable unfolding and enrichment of the science of mathematics" (Zermelo, 1930).

CONCLUSION

- Infinite division indicates only the appearance as quantum continuum and is inseparable from the filling of space (MATTER). The ground of matter's infinite divisibility lies precisely in that.

- But as soon as something is assumed as a quantum discretum the multiplicity (*Menge*) of units in it is determined; hence it is always equal to a number (there is always a smallest unit found in the progressive synthesis of a series in dividing matter and in composing conditions)

- Thus only experience can settle how far the organization in an articulated body may go (Kant, KrV A527/B555)

(Further research on Kant's philosophy of mathematics and conception of geometry is needed).

CONCLUSION

- By modifying Descartes and Baumgarten distinction between regressum in infinitum and in indefinitum, Kant introduced and applied concepts of metaphysics and mathematics to his transcendental philosophy
- The notion of regressum in indefinitum is crucial to reconcile his cosmology and the Critique of pure Reason
- Progress in infinitum and in indefinitum are both allowed, but the regress can be only in indefinitum in dealing with quanta continua (matter, aether filling the universe is continuous)
- Every series which is dabilis (or given) must be an empirical one vs. non dabilis series

CONCLUSION

- Notion of non datum (it could also be given in the future or be simply unknown now, but not in the future). Kant limits the rights of logic and metaphysics to infer from the mere possibility the existence of a member of the series or of a «not-yet-given» unit in a multiplicity which is unbounded.
- Distinction between a regressum which has to do with decomposition or division and the regress of composition or connection

e.g. Distiction to be made also between a progressum in infinitum and ad infinitum that both determine the magnitude IN the object, but the former does so in the act of dividing, whereas the latter in the act of composing conditions.

THANK YOU

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