The Pascalian Notion of Infinity
– what does "infinite distance" mean?

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Aims

- Main goal: to compare the notion of **infinite distance** in Pascal's projective geometry works and in the apologetic context of *Pensées*
- **Heterogeneity** in mathematical works and in *Pensées* (Gardies 1984, Magnard 1992)

Blaise Pascal (1623-1662)

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- To analyze Pascal's contribution to the XVIIth century discussion on infinity. If on the one hand Pascal accepts Nature as infinite, on the other hand he does a **negative use** of infinity – this is reflected in the notion of *infinite distance*

Methodological remarks

- I do not intend to say that the fragments of *Pensées* can be perfectly translated into mathematical language. Instead, what I am inquiring into is about abstract models from which both Pascal's mathematic works and his apologetics could share common features (Serres 1968)

- Pascal does not define *infinity* or *infinite distance*. Differently from other philosophers of the XVIIth century, Pascal does not have a philosophical system and neither an explicit definition of his concepts

- I refer to Pascal's and Desargues' works as in the domain of "projective geometry", even if there are controversies about whether we can call XVIIth century works properly as projective geometry works

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Content

Fundamental passages of *Pensées*

"Let us therefore examine this point, and say: God is, or is not. But towards which side will we lean? Reason cannot decide anything here. There is an **infinite chaos** separating us. At the extremity of this **infinite distance** a game is being played and the coin will come down heads or tails. What will you wager? Reason cannot make you choose one way or the other; reason cannot make you defend either of the two choices" (Sel. 680, Laf. 418).

"The **infinite distance** between body and mind [*esprit*] points to [*figure*] the **infinitely more infinite distance** between mind and charity; for it is supernatural" (Sel. 339, Laf. 308).

Fundamental passages of *Pensées*

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Disproportion of man (Sel. 230, Laf. 199)

"What is man in infinity? (...) For in the end what is man in
nature? A nothingness compared to the infinite, everything
compared to a nothingness, a mid-point between nothing and
everything, **infinitely far** [*infiniment éloigné*] from
comprehending the extremes; (...) Within the scope of this
infinities all finites are equal, and I do not see why we settle
our imagination on one rather than the other. Simply
comparing ourselves to the finite distresses us".

Since the Fall, man is lost between the infinity of greatness
and infinity of smallness. Pascal sees **a relation between
the two infinities**

There is no type of proportion that can measure man in
relation to anything within nature

Pascal's works on projective geometry

- *L'essai pour les coniques* (1640)

- The lost *Traité des coniques*. What lasted:
  - Letter to the *Celleberrimae Matheseos Academiae Parisiensi* (1654)
  - Letter from Leibniz to Étienne Périer (1676)
  - *Generatio Conisectionum*

Influences: Desargues' projective geometry

- March 1639: *Brouillon project d'une Atteinte aux evenemens des rencontres du cone avec un plan*
- Baroque writing style – severely criticized
- Introduction of elements at infinite distance
- Mesnard (1994): Desargues created a "geometry of infinity" when he applied the idea of infinity to pure geometry
- Parallel lines meet at infinity
- The extremities of a line meet at infinity
- Elements at infinity allowed a generalization of the study of the conic sections
- Desargues was the first to treat points at infinity as entirely ordinary points, even if Kepler had also introduced points at infinity (Field 1994)

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1639.   AVEC PRIVILEGE.

BROVILLON PROIECT D'VNE ATTEINTE AVX
euenemens des rencontres du Cone auec vn Plan, Par L, S, G, D, L.

Noms impofit̃,

Ine fera pas malaifé de faire icy la diftinction neceffaire d'entre les impofitions de nom, autrement definitions, les propofitions, les demonftrations, quand elles font en fuitte. Et les autres efpeces de difcours non plus que de choifir entre les figures celle qui à raport au periode qu'on lit, ou de faire ces figures fur le difcours.

Chacun penfera ce qui luy femblera conuenable ou de ce qui eft icy deduit, ou de la maniere de le déduire, & verra que la raifon effaye à cognoiftre des quantitez infinies d'vne part: enfemble de celles qui s'apetiffent iufques à reduire leurs deux extremitez oppofées en vne feule, & que l'entendement s'y pert, non feulement à caufe de leurs imaginables grandeur & petiteffe, mais encore à caufe que le raifonnement ordinaire le conduit à en conclure des propietez, d'où il eft incapable de comprendre comment, c'eft qu'elles font.

Icy toute ligne droicte eft entendu'alongée au befoin à l'infiny d'vne part & d'autre.

Vn femblable alongement à diftance infinie d'vne part & d'autre en vne droicte, eft icy reprefenté par vne rangée de poincts alignez d'vne part & d'autre en fuitte de cette droicte.

Pour donner à entendre de plufieurs lignes droictes, qu'elles font toutes entre-elles où bien paralleles, où bien inclinées à mefme poinct. Il eft icy dict, que toutes ces droictes font d'vne mefme ordonnance entre elles, par où l'on conceura de ces plufieurs droictes, qu'en l'vne auffi bien qu'en l'autre de ces deux efpece de pofition elles tendent toutes à vn mefme endroict.

Ordonnance de lignes droictes.

L'endroict auquel on conçoit que tendent ainfi plufieurs droictes en l'vne auffi bien qu'en l'autre de ces deux efpeces de pofition, eft icy nommé, *but*, de l'ordonnance de ces droictes.

But, d'vne ordonnance de droictes.

Pour donner à entendre l'efpece de pofition d'entre plufieurs droictes en laquelle elles font toutes paralleles entre elles, il eft icy dict, que toutes ces droictes font entre elles d'vne mefme ordonnance, dont le but eft à diftance infinie en chacune d'vne part & d'autre.

Pour donner à entendre l'efpece de pofition d'entre plufieurs droictes, en laquelle elles font toutes inclinées à vn mefme poinct, il eft icy dict, que toutes ces droictes font entre elles d'vne mefme ordonnance, donte le but eft à diftance finie en chacune d'elles.

Ainfi deux quelconques droictes en vn mefme Plan, font entre elles d'vne mefme ordonnan-ce, dont le but eft à diftance ou finie, ou infinie.

Icy tout Plan eft entendu pareillement eftendu de toutes parts à l'infiny.

Vn femblable eftendu d'vn Plan à l'infini de toutes parts, eft icy reprefenté par vn nombre de poincts femez de toutes parts aux extremitez du Plan.

Pour donner à entendre de plufieurs Plans, qu'ils font tous entre eux ou bien paralels, ou bien inclinez à vne mefme droicte, il eft icy dict, que tous ces Plans font entre eux d'vne mef-me, *ordonnance*, par où l'on conceura de ces plufieurs Plans qu'en l'vne auffi bien qu'en l'autre de ces deux efpeces de pofition, ils tendent tous à vn mefme endroict.

Ordonnance de Plans.

L'endroict auquel on conçoit que tendent ainfi plufieurs Plans en l'vne auffi bien qu'en l'au-tre de ces deux efpeces de pofition, eft icy nommé, *but*, de l'ordonnance de ces Plans.

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Some elements of Pascal's projective geometry

All conic sections are considered as projections of the circle.

Pascal exposes six kinds of conic sections: point, line, angle, antobola [ellipse], parabola, hyperbola

Generatio conisectionum (OC, II, pp. 1008-1119)

- 3rd definition: Two or more lines are concurrent if they intersect at the same point, at finite or infinite distance [*ad distantiam vel finitam, [...] vel infinitam*]

- 2nd definition: A line is said to tend towards a point when, being sufficiently extended, it reaches that point; and a line is said to be conducted or to tend towards a point on another line at infinite distance [*ad distantiam infinitam*] when it is parallel to the second line

- 5th definition: An infinite line conducted on the plane of a conic, that does not meet the conic except at an infinite distance [*nisi ad distantiam infinitam*] (...) is said to be an asymptote

- Asymptotes should be considered as tangents at an infinite distance [*ad distantiam infinitam*]

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An infinite closed line

- In Desargues' projective geometry, the two extremities of an infinite line meet one another

- Pascal appropriates this idea in *Pensées*: "These extremes touch and join by force of distance, meeting in God and God alone" (Sel. 230, Laf. 199). The point where both infinities meet is the position of God

- *Pensées*: "Jesus Christ is the object of everything, and the centre to which everything tends. Whoever knows him knows the reason for everything" (Sel. 690, Laf. 449).

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Heterogeneity

Book V of the *Elements*, definitions 3 and 4
Eudoxus Axiom / Archimedian Property

Pascal, *De l'esprit géométrique*: "Euclid defines in this way the homogeneous magnitudes: 'Magnitudes, he says, are said to be of the same genre, when one of them is capable, when several times multiplied, of exceeding the other"'(OC, III, p. 408).

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Heterogeneity in Pascal's mathematical works

"In this way, points do not add anything to lines, lines to areas, areas to solids, or, *to employ the language of numbers in a treatise devoted to numbers*, the roots do not count in relation to squares, squares to cubes, cubes to *carro-carrés*, etc. Thus the lower degrees should be neglected as lacking any value" (*Potestatum numericarum summa*).

Different orders in Pensées

- "Out of all bodies together we could not succeed in making one little thought. It is impossible, and of another order. Out of all bodies and minds we could not draw one impulse of true charity. It is impossible, and of another, supernatural, order" (Sel. 339, Laf. 308).

- The difference between the three orders bears a resemblance to mathematical heterogeneity: all elements of a lower order are nothing with regard to those of a higher one

- One can think about different orders of infinity

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Pascal and the actual infinite

Cantor attributed to Pascal one of the first attempts to make explicit the concept of actual infinity

"We know that there is an infinite, but we do not know its nature; as we know that it is false that numbers are finite, so therefore it is true that there is an infinite number, but we do not know what it is: it is false that it is even and false that it is odd, for by adding a unit it does not change its nature; however it is a number, and all numbers are even or odd (it is true that this applies to all finite numbers).
So we can clearly understand that there is a God without knowing what he is." (Sel. 680, Laf. 418)

A boat going to the horizon

- *De l'esprit géométrique*: If we regard with a telescope a boat that always goes away, it never arrives to the horizon, and its image never arrives at the extremity of the telescope

- Relation between the infinity of greatness and the infinity of smallness

- Relation between infinity and the infinitely far

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Summarising

- Projective geometry's elements at infinity (referred to as at *infinite distances*) seem to relate to Pascal's notion of infinite distance in *Pensées*

- The notion of infinite distance in *Pensées* also bears a relation to the concept of *heterogeneity*, on which Pascal reflected later on his works

- While infinite distance in projective geometry allows a generalization in the study of the conic sections, infinite distance in *Pensées* is a *figure* of disproportion and of heterogeneity

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Infinity for Pascal and the context of the XVIIth century

- XVIIth century discussions on infinity as an attribute of God, as the possible size of the universe and as the infinitely small in mathematics

- For Pascal, the notion of an *infinite distance* allows generalizations in projective geometry with the introduction of new elements

- **But in apologetics infinity has also a negative use**: man is at an *infinite distance* from any reference outside God, the three orders are infinitely distant from one another

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Thank you

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