Dialogic Logic (1).

- Two players, the **Proponent** and the **Opponent**.
- In the round 0, the Proponent has to assert the formula to be proved and the Opponent can make as many assertions as he wants. After that, the opponent starts the game.
- In all other moves, the players have to do an *announcement* and an *action*.
- An **announcement** is either of the form $\mathbf{attack}(n)$ or of the form $\mathbf{defend}(n)$, interpreted as “I shall attack the assertion made in round n” and “I shall defend myself against the attack made in round n”.
- An **action** can be one of the following moves: $\mathbf{assert}(\Phi)$, $\mathbf{which\ one}?$, $\mathbf{left}?$, $\mathbf{right}?$, $\mathbf{what\ if}?\mathbf{assert}(\Phi)$.
- You can only attack lines in which the other player asserted a formula.
  Depending on the formula, the following attacks are allowed:
  - $\Phi \vee \Psi$ may be attacked by $\mathbf{which\ one}?$,
  - $\Phi \wedge \Psi$ may be attacked by $\mathbf{left}?$ or $\mathbf{right}?$,
  - both $\Phi \to \Psi$ and $\neg\Phi$ may be attacked by “$\mathbf{what\ if}?, \mathbf{assert}(\Phi)$”.

Dialogic Logic (2).

- You can only defend against a line in which the other player attacked.
  Depending on the attack, the following defenses are allowed:
  - If $\Phi \vee \Psi$ was attacked by $\mathbf{which\ one}?$, you may defend with either $\mathbf{assert}(\Phi)$ or $\mathbf{assert}(\Psi)$.
  - If $\Phi \wedge \Psi$ was attacked by $\mathbf{left}?$, you may defend with $\mathbf{assert}(\Phi)$, if it was attacked by $\mathbf{right}?$, you may defend with $\mathbf{assert}(\Psi)$.
  - If $\Phi \rightarrow \Psi$ was attacked by "$\mathbf{what\ if}?, \mathbf{assert}(\Phi)$", you may defend with $\mathbf{assert}(\Psi)$.
  - You cannot defend an attack on $\neg\Phi$.

Dialogic Logic (3).

The rules of the constructive game:

- In each move, the action and the announcement have to fit together, i.e., if the player announces $\mathbf{attack}(n)$ or $\mathbf{defend}(n)$, then the action has to be an attack on move n or a defense against move n.
- In round $n+1$, the Opponent has to either attack or defend against round n.
- An attack is called open if it has not yet been defended.
- The Proponent may attack any round, but may only defend against the most recent open attack. He may use any defense or attack against a given round at most once.
- The Opponent may assert any atomic formulas.
- The Proponent may assert only atomic formulas that have been asserted by the Opponent before.

Dialogic Logic (3).

The rules of the classical game:

- In each move, the action and the announcement have to fit together, i.e., if the player announces $\mathbf{attack}(n)$ or $\mathbf{defend}(n)$, then the action has to be an attack on move n or a defense against move n.
- In round $n + 1$, the Opponent has to either attack or defend against round n.
- An attack is called open if it has not yet been defended.
- The Proponent may attack and defend against any round. He may use any defense or attack against a round at most once.
- The Opponent may assert any atomic formulas.
- The Proponent may assert only atomic formulas that have been asserted by the Opponent before.

Dialogic logic (4).

We say that Φ is **(dialogically/classically) valid** if the Proponent has a winning strategy in the (constructive/classical) game in which he asserts Φ in round 0.

Example.

0		—		$\mathbf{assert}(\neg\neg p \to p)$
1	$\mathbf{attack}(0)$	$\mathbf{what\ if?}\ \mathbf{assert}(\neg\neg p)$		
2			$\mathbf{attack}(1)$	$\mathbf{what\ if?}\ \mathbf{assert}(\neg p)$
3	$\mathbf{attack}(2)$	$\mathbf{what\ if?}\ \mathbf{assert}(p)$		
4			—	—

Dialogic logic (4).

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Example.

0		—		$\mathbf{assert}(\neg\neg p \to p)$
1	$\mathbf{attack}(0)$	$\mathbf{what\ if}?\ \mathbf{assert}(\neg\neg p)$		
2			$\mathbf{attack}(1)$	$\mathbf{what\ if}?\ \mathbf{assert}(\neg p)$
3	$\mathbf{attack}(2)$	$\mathbf{what\ if}?\ \mathbf{assert}(p)$		
4			$\mathbf{defend}(1)$	$\mathbf{assert}(p)$
5	—	—		

Obligationes (1).

Obligationes. A game-like disputation, somewhat similar to logic games. The origin is unclear, as is the purpose.
The name derives from the fact that one of the players is "obliged" to follow certain formal rules of discourse.

Different types of *obligationes*.

- *positio*.
- *depositio*.
- *dubitatio*.
- *impositio*.
- *petitio*.
- *rei veritas / sit verum*.

Obligationes (2).

- William of Shyreswood (1190-1249)
- Walter Burley (Burleigh; c.1275-1344)
- Roger Swyneshed (d.1365)
- Richard Kilvington (d.1361)
- William Ockham (c.1285-1347)
- Robert Fland (c.1350)
- Richard Lavenham (d.1399)
- Ralph Strode (d.1387)
- Peter of Candia
- Paul of Venice (c.1369-1429)

Obligationes (3).

- Walter Burley, *De obligationibus*.
  Standard set of rules.
- Roger Swyneshed, *Obligationes* (1330-1335).
  Radical change in one of the rules results in a distinctly different system.

responsio antiqua	*responsio nova*
Walter Burley	Roger Swyneshed
William of Shyreswood	Robert Fland
Ralph Strode	Richard Lavenham
Peter of Candia	
Paul of Venice	

positio according to Burley (1).

- Two players, the **opponent** and the **respondent**.
- The **opponent** starts by positing a *positum* φ^*.
- The **respondent** can "admit" or "deny". If he denies, the game is over.
- If he admits the *positum*, the game starts. We set $\Phi_0 := \{\varphi^*\}$.
- In each round n, the **opponent** proposes a statement φ_n and the **respondent** either "concedes", "denies" or "doubts" this statement according to certain rules. If the **respondent** concedes, then $\Phi_{n+1} := \Phi_n \cup \{\varphi_n\}$, if he denies, then $\Phi_{n+1} := \Phi_n \cup \{\neg\varphi_n\}$, and if he doubts, then $\Phi_{n+1} := \Phi_n$.

positio according to Burley (2).

- We call φ_n pertinent (relevant) if either $\Phi_n \vdash \varphi_n$ or $\Phi_n \vdash \neg\varphi_n$. In the first case, the **respondent** has to concede φ_n, in the second case, he has to deny φ_n.
- Otherwise, we call φ_n impertinent (irrelevant). In that case, the **respondent** has to concede it if he knows it is true, to deny it if he knows it is false, and to doubt it if he doesn't know.
- The **opponent** can end the game by saying *Tempus cedat*.

Example 1.

Opponent	Respondent	
I posit that Cicero was the teacher of Alexander the Great: φ^*.	I admit it.	$\Phi_0 = \{\varphi^*\}$.
Cicero was Roman: φ_0.	I concede it.	Impertinent and true; $\Phi_1 = \{\varphi^*, \varphi_0\}$.
The teacher of Alexander the Great was Roman: φ_1.	I concede it.	Pertinent, follows from Φ_1.

Example 2.

Opponent	Respondent	
I posit that Cicero was the teacher of Alexander the Great: φ^*.	I admit it.	$\Phi_0 = \{\varphi^*\}$.
The teacher of Alexander the Great was Greek: φ_0.	I concede it.	Impertinent and true; $\Phi_1 = \{\varphi^*, \varphi_0\}$.
Cicero was Greek: φ_1.	I concede it.	Pertinent, follows from Φ_1.

Example 3 (“order matters!”)

Opponent	Respondent	
I posit that Cicero was the teacher of Alexander the Great: φ^*.	I admit it.	$\Phi_0 = \{\varphi^*\}$.
The teacher of Alexander the Great was Roman: φ_0.	I deny it.	Impertinent and false; $\Phi_1 = \{\varphi^*, \neg\varphi_0\}$.
Cicero was Roman: φ_1.	I deny it.	Pertinent, contradicts Φ_1.

Properties of Burley's *positio*.

- Provided that the *positum* is consistent, no disputation requires the **respondent** to concede φ at step n and $\neg\varphi$ at step m.
- Provided that the *positum* is consistent, Φ_i will always be a consistent set.
- It can be that the **respondent** has to give different answers to the same question (Example 4).
- The **opponent** can force the **respondent** to concede everything consistent (Example 5).

Example 4.

Suppose that the **respondent** is a student, and does not know whether the King of France is currently running.

Opponent	**Respondent**	
I posit that you are the Pope or the King of France is currently running: φ^*	I admit it.	$\Phi_0 = \{\varphi^*\}$.
The King of France is currently running: φ_0 .	I doubt it.	Impertinent and unknown; $\Phi_1 = \{\varphi^*\}$.
You are the Pope: φ_1.	I deny it.	Impertinent and false; $\Phi_2 = \{\varphi^*, \neg\varphi_1\}$.
The King of France is currently running: $\varphi_2 = \varphi_0$.	I concede it.	Pertinent, follows from Φ_2.

Example 5.

Suppose that φ does not imply $\neg\psi$ and that φ is known to be factually false.

Opponent	**Respondent**	
I posit φ.	I admit it.	$\Phi_0 = \{\varphi\}$.
$\neg\varphi \vee \psi$.	I concede it.	Either φ implies ψ, then the sentence is pertinent and follows from Φ_0; or it doesn't, then it's impertinent and true (since φ is false); $\Phi_1 = \{\varphi, \neg\varphi \vee \psi\}$.
ψ	I concede it.	Pertinent, follows from Φ_1.

positio according to Swyneshed.

- All of the rules of the game stay as in Burley's system, except for the definition of *pertinence*.
- In Swyneshed's system, a proposition φ_n is pertinent if it either follows from φ^* (then the **respondent** has to concede) or its negation follows from φ^* (then the **respondent** has to deny). Otherwise it is impertinent.

Properties of Swyneshed's *positio*.

- Provided that the *positum* is consistent, no disputation requires the **respondent** to concede φ at step n and $\neg\varphi$ at step m.
- The **respondent** never has to give different answers to the same question.
- Φ_i can be an inconsistent set (Example 6).

Example 6.

Suppose that the respondent is a student in Paris, and not a bishop. Write ψ_0 for "You are in Rome" and ψ_1 for "You are a bishop".

Opponent	**Respondent**	
I posit that you are in Rome or you are a bishop: $\psi_0 \vee \psi_1$	I admit it.	$\Phi_0 = \{\psi_0 \vee \psi_1\}$.
You are in Rome or you are a bishop: $\psi_0 \vee \psi_1$.	I concede it.	Pertinent, follows from Φ_0; $\Phi_1 = \{\psi_0 \vee \psi_1\}$.
You are not in Rome: $\neg\psi_0$.	I concede it.	Impertinent, and true; $\Phi_2 = \{\psi_0 \vee \psi_1, \neg\psi_0\}$.
You are not a bishop: $\neg\psi_1$.	I concede it.	Impertinent, and true; $\Phi_3 = \{\psi_0 \vee \psi_1, \neg\psi_0, \neg\psi_1\}$.

Φ_2 is an inconsistent set of sentences.

positio according to Kilvington.

Richard Kilvington (d.1361).

- *Sophismata*, c.1325.
- *obligationes* as a solution method for sophismata.
- He follows Burley's rules, but changes the handling of impertinent sentences. If φ_n is impertinent, then the **respondent** has to concede if it were true if the *positum* was the case, and has to deny if it were true if the *positum* was not the case.

impositio.

- In the *impositio*, the **opponent** doesn't posit a *positum* but instead gives a definition or redefinition.
- **Example 1.** "In this *impositio*, *asinus* will signify *homo*".
- **Example 2.** "In this *impositio*, *deus* will signify *homo* in sentences that have to be denied or doubted and *deus* in sentences that have to be conceded."

  Suppose the **opponent** proposes "*deus est mortalis*".

  - If the **respondent** has to deny or doubt the sentence, then the sentence means *homo est mortalis*, but this is a true sentence, so it has to be conceded. Contradiction.
  - If the **respondent** has to concede the sentence, then the sentence means *deus est mortalis*, but this is a false sentence, so it has to be denied. Contradiction.
- An *impositio* often takes the form of an insoluble.

1400-1550.

- **1453.** The Fall of Constantinople.
- **c. 1400-1468.** Johannes Gutenberg.
- **1492.** The Discovery of the Americas.
- **1483-1546.** Martin Luther.

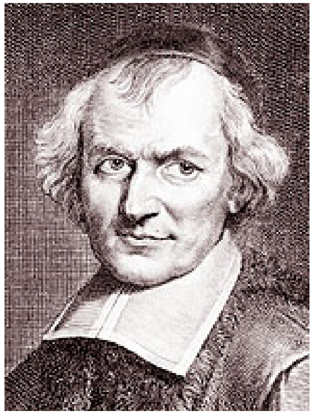
Pierre de la Ramée.

Pierre de la Ramée (Petrus Ramus; 1515-1572)

- *Animadversiones in Dialecticam Aristotelis* (1543).
- Professor at the *Collège de France*.
- Ramistic Logic. *ars disserendi*. Logic of natural discourse.
- Protestant. Died in the Massacre of St. Bartholomew (August 24th, 1572).

Port Royal.

- Cornelius Jansen (1585-1638), bishop of Ypres; *Augustinus* (1640), doctrine of strict predestination.
- Abbey of Port Royal, since 1638 centre of Jansenism.
- Pierre Nicole (1625-1695); Antoine Arnauld (1612-1694)



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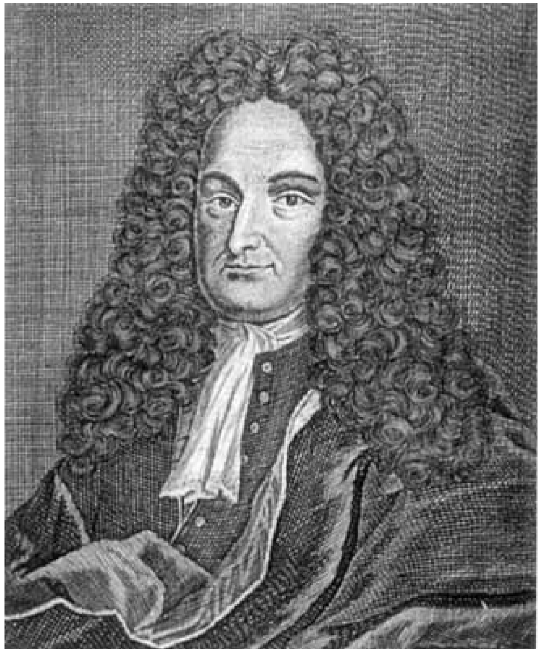
- **1662.** *La logique, ou l'art de penser*. Opposing scholasticism, "epistemological turn".
- *Comprehension* vs *Extension*.
- Letters between Arnauld and Leibniz: 1687-1690.

Comprehension vs Extension.

- *Comprehension of X.* The set of properties that x has to have in order to be an X.
- *Extension of X.* The set of all X.
- *An example.*
  - Universe of Discourse: $U = \{a, A, A, B, b\}$
  - Properties: **Consonant**, **Capital**, **Blue**.
  - Extensions:
    - **Consonant** $\rightsquigarrow \{B, b\}$
    - **Capital** $\rightsquigarrow \{A, A, B\}$
    - **Blue** $\rightsquigarrow \{a, B, b\}$
  - The Comprehension of **Consonant** in this universe of discourse includes the property **blue**.

Leibniz (1).



Gottfried Wilhelm von Leibniz (1646-1716)

- Work on philosophy, mathematics, law (Doctorate in Law from the University of Altdorf (1667), alchemy, theology, physics, engineering, geology, history.
- Diplomatic tasks (1672).
- Attempts to build a calculating machine (1670s).

Leibniz (2).

- **1673-1677**: Invented calculus independently of Sir Isaac Newton (1643-1727).
- Research politics; foundation of Academies: Brandenburg, Dresden, Vienna, and St Petersburg.
- **1710**: *Théodicée*. “The best of all possible worlds”.

Leibniz (3).

Properties.

- *Identity of Indiscernibles:* If $\{\Phi\,;\,\Phi(x)\} = \{\Phi\,;\,\Phi(y)\}$, then $x = y$.
- Primary substances ("Plato", "Socrates") can be expressed in terms of properties: a uniform language of predication.
- Connected to Leibniz' monadology (1714).

Relations.

- Call for an analysis of relations.
- Attempt to reduce relations to unary predicates:

"Plato is taller than Socrates"	$\mathbf{Taller}(\mathrm{Pla}, \mathrm{Soc})$
"Plato is tall *in as much as* Socrates is short"	$\mathbf{Tall}(\mathrm{Pla}) \oplus \mathbf{Short}(\mathrm{Soc})$

Calculemus!

*“quando orientur controversiae, non magis disputatione opus erit inter duos philosophos, quam inter duos Computistas. Sufficiet enim calamos in manus sumere sedereque ad abacos, et sibi mutuo (accito si placet amico) dicere: **calculemus**.”*

$\rightsquigarrow$ Arithmetization of Language and Automatization of Reasoning

Arithmetization of Language (1).

- *characteristica universalis*: general notation system for everything, based on the unanalyzable basics.
- *calculus ratiocinator*: formal system with a mechanizable deduction system.
- "*calculus de continentibus et contentis est species quaedam calculi de combinationibus*"
- The properties correspond to the natural numbers $n > 1$. The unanalyzable properties correspond to the prime numbers.
- **Example.** If *animal* corresponds to 2, and *rationalis* corresponds to 3, then *homo* would correspond to 6. If *philosophicus* corresponds to 5, then *philosophus* = *homo philosophicus* would be 30.

Arithmetization of Language (2).

animal $\rightsquigarrow 2$, *rationalis* $\rightsquigarrow 3$, *homo* $\rightsquigarrow 6$, *philosophicus* $\rightsquigarrow 5$, *philosophus* $\rightsquigarrow 30$.

- All individuals are determined by their properties, so *Socrates* is represented by a number n. Since Socrates is a philosopher, $30|n$.
- In general, "the individual represented by n has the property represented by m" is rendered as $m|n$.
- Now we can formalize $A\mathrm{a}B$ and $A\mathrm{i}B$. Let n_A and n_B be the numbers representing A and B, respectively.
  - $A\mathrm{a}B$: $n_A|n_B$.

    "Every human is an animal": $2|6$.
  - $A\mathrm{i}B$: $\exists k(n_A|k \cdot n_B)$.

    "Some human is a philosopher": $30|5 \cdot 6$.

Arithmetization of Language (3).

$A\mathrm{a}B$: $n_A | n_B$; $A\mathrm{i}B$: $\exists k(n_A | k \cdot n_B)$.

- **Barbara** becomes: "If $n|m$ and $m|k$, then $n|k$."
  So, the laws of arithmetic prove **Barbara**.
- **Darii** becomes: "If $n|m$ and there is some w such that $m|w \cdot k$, then there is some w^* such that $n|w^* \cdot k$."
  (Let $w^* := w$.)
- **But:** $A\mathrm{i}B$ is always true, as $n|n \cdot m$ for all n and m.
- If n represents *homo* and m represents *asinus*, then $n \cdot m$ would be a "man with the added property of being a donkey".
- This simple calculus is not able to deal with negative propositions.