UNIVERSITEIT VAN AMSTERDAM
INSTITUTE FOR LOGIC, LANGUAGE AND COMPUTATION

Core Logic

2006/2007; 1st Semester
dr Benedikt Löwe

Homework Set # 9 *Deadline:* November 22nd, 2006

Exercise 29 (6 points).
A structure $\langle R, +, \cdot, 0, 1\rangle$ is called a **ring** if $+$ is a commutative and associative binary operation on R, $\cdot$ is an associative binary operation on R, $\cdot$ distributes over $+$ (*i.e.*, $a \cdot (b + c) = a \cdot b + a \cdot c$ and $(a + b) \cdot c = a \cdot c + b \cdot c$), 0 is the neutral element of $+$ (*i.e.*, $0 + a = a + 0 = a$) and 1 is the neutral element of $\cdot$ (*i.e.*, $a \cdot 1 = 1 \cdot a = a$).
Examples of rings are: the integers $\mathbb{Z}$, the rationals $\mathbb{Q}$, the reals $\mathbb{R}$.

Let $\mathbf{B} = \langle B, 0, 1, \vee, \wedge, -\rangle$ be a Boolean algebra. For $X, Y \in B$, define

$$X + Y := (X \wedge -Y) \vee (-X \wedge Y), \text{ and}$$

$$X \cdot Y := X \wedge Y.$$

We write $\mathrm{R}(\mathbf{B}) := \langle B, +, \cdot, 0, 1\rangle$.

(1) Prove that $\mathrm{R}(\mathbf{B})$ is a ring (3 points).
(2) Give an example of a ring R such that R is not isomorphic to any $\mathrm{R}(\mathbf{B})$ (with a proof; 3 points).

Exercise 30 (6 points).

Let $\mathcal{L} = \{R\}$ be a language with one binary relation symbol. Consider the following seven $\mathcal{L}$-sentences:

$$\begin{aligned}
\varphi_{(\mathrm{i})} &:\leftrightharpoons \forall x \neg Rxx \\
\varphi_{(\mathrm{ii})} &:\leftrightharpoons \forall x \forall y (x \neq y \to (Rxy \vee Ryx)) \\
\varphi_{(\mathrm{iii})} &:\leftrightharpoons \forall x \forall y \forall z ((Rxy \wedge Ryz) \to Rxz) \\
\varphi_{(\mathrm{iv})} &:\leftrightharpoons \forall x \exists y \exists z (Ryx \wedge Rxz) \\
\varphi_{\mathrm{ME}} &:\leftrightharpoons \exists x \forall y (Ryx \vee x = y) \\
\varphi_{\mathrm{LEP}} &:\leftrightharpoons \forall x \exists y \forall z (Rxz \to (Rzy \vee y = z))
\end{aligned}$$

Check whether the following sets of sentences are consistent. If they are, give a model. If they aren't, derive a contradiction (2 points each).

(1) $\{\varphi_{(\mathrm{i})}, \varphi_{(\mathrm{iii})}, \varphi_{(\mathrm{iv})}, \varphi_{\mathrm{ME}}\}$,
(2) $\{\varphi_{(\mathrm{i})}, \varphi_{(\mathrm{iii})}, \varphi_{\mathrm{LEP}}, \neg\varphi_{\mathrm{ME}}\}$,
(3) $\{\varphi_{(\mathrm{i})}, \varphi_{(\mathrm{ii})}, \varphi_{(\mathrm{iii})}, \varphi_{\mathrm{LEP}}, \neg\varphi_{\mathrm{ME}}\}$,

Exercise 31 (6 points).

It is often said that Frege was an anti-semite. What is our source for this claim? Please describe the source with full bibliographical data (2 points).

Frege's anti-semitism is often contrasted with his work in logic. He is sometimes mentioned as an example for a researcher with unacceptable political views that had no influence on his research. Find a scholarly source for a claim like this (2 points).

A famous Frege expert X was shocked when he find out that Frege held anti-semitic opinions. A reviewer Y of X's major work on Frege writes "This was a great shock for X but not for me". Y had gathered from Frege's choice of example sentences in his logical texts that he was very much in line with late XIXth century German conservative thought. Who are X and Y? (1 point each; give bibliographic references to Y's review and the 'major work' of X)

(In this exercise, all citations have to be sources that could also be cited in a historical research paper, i.e., published books or journal articles in respectable scientific journals. Quoting `wikipedia.org` *or some webpage is not acceptable and will give no credit.)*

Exercise 32 (4 points).

In September 1889, a number of German mathematicians met in Heidelberg at the meeting of the *Gesellschaft Deutscher Naturforscher und Ärzte* to discuss the plans to form a German mathematical society. Georg Cantor and Walther Dyck played an important role during these discussions. Who was the chairman of this meeting (during which 15 mathematicians were present)? (1 point)

A year later (1890), the German mathematicians met in Bremen and prepared the foundation of the *Deutsche Mathematiker-Vereinigung* (DMV) in the so-called *Bremer Beschlüssen*. How many mathematicians signed this document? (1 point)

Another year later, the DMV held its first meeting in Halle very successfully, but the 1892 meeting (planned for Nuremberg) was cancelled. The Nuremberg cancellation was a major factor in the animosities between Cantor and Dyck. Explain in a few sentences what happened and why Cantor was angry at Dyck (2 points).