UNIVERSITEIT VAN AMSTERDAM
INSTITUTE FOR LOGIC, LANGUAGE AND COMPUTATION

Core Logic

2006/2007; 1st Semester
dr Benedikt Löwe

Homework Set # 7 *Deadline:* November 8th, 2006

Exercise 22 (7 points).

We are considering two new systems of dialogic logic: In the first one, called **strictly constructive**, we restrict the proponent in a way that he also can only react to the last move of the opponent and denote the corresponding semantic relation by $\models_{\mathrm{sc}}$. In the second one, called **liberal**, $\models_{\mathrm{lib}}$, we liberalize the opponent so that he also can react to all prior moves of the proponent.

(1) Give formal definitions (in the style of the lecture, giving explicitly the rules for the two players) for $\models_{\mathrm{sc}}$ and $\models_{\mathrm{lib}}$ (½ points each).
(2) Prove that $\models_{\mathrm{lib}} \varphi$ holds for no formula φ (2 points).
(3) Find two different formulas φ such that $\models_{\mathrm{sc}} \varphi$ and give dialogue proofs for them (1 point each).
(4) Find a formula φ such that $\models_{\mathrm{dialog}} \varphi$ but not $\models_{\mathrm{sc}} \varphi$. Give proofs of both claims (1 point each).

Exercise 23 (4 points).

Give dialogue proofs of the following formulas in $\models_{\mathrm{cl}}$ (1 point each):

- $\neg\neg\neg p \rightarrow \neg p$,
- $((p \rightarrow q) \wedge \neg q) \rightarrow \neg p$.

For both formulas, decide whether they are valid in $\models_{\mathrm{dialog}}$ and give a dialogue argument for or against your claim (1 point each).

Exercise 24 (6 points).

In this exercise, we consider the systems of *positio* as described by Walter Burley and Roger Swyneshed. If a *positum* φ^* is given and φ_k (for $0 \leq k \leq n$) are proposed sentences of the **Opponent**, we let Φ_k^{Burley} be the set of **"currently accepted truths"** according to Burley's system on the basis of the sequence $\langle \varphi^*, \varphi_0, \ldots, \varphi_n \rangle$.
Prove the following properties of the two systems:

(1) If the *positum* φ^* is consistent, then for all $k \leq n$, the set Φ_k^{Burley} is a consistent set (3 points).
(2) If the *positum* φ^* is consistent and $k < \ell \leq n$ with $\varphi_k = \varphi_\ell$, then the **Respondent** in a Swyneshed-style *positio* will give the same answer in steps k and ℓ of the *obligatio* (3 points).