UNIVERSITEIT VAN AMSTERDAM
INSTITUTE FOR LOGIC, LANGUAGE AND COMPUTATION

Core Logic

2006/2007; 1st Semester
dr Benedikt Löwe

Homework Set # 4 *Deadline:* October 4th, 2006

Exercise 11 (8 points).

Let Believe, Fear, and Doubt be operators corresponding to the natural language expressions "I believe", "I fear", and "I doubt", *i.e.*, the meaning of $\mathsf{Fear}(p)$ is "I fear that p", *etc.*.
Give examples (in terms of a little story that provides the necessary background information required for evaluating the natural language expressions) for the **in**validity of the following rules (2 points each):

- If $\mathsf{Believe}(p \vee q)$, then $\mathsf{Believe}(p) \vee \mathsf{Believe}(q)$.
- If $\mathsf{Fear}(p \wedge q)$, then $\mathsf{Fear}(p) \wedge \mathsf{Fear}(q)$.
- If $\mathsf{Doubt}(p \wedge q)$, then $\mathsf{Doubt}(p) \wedge \mathsf{Doubt}(q)$.
- If $\mathrm{Fear}(\neg p)$, then not $\mathrm{Fear}(p)$.

Exercise 12 (8 points).

Read

> Alan **Code**, Aristotle's response to Quine's objections to modal logic, **Journal of Philosophical Logic** 5 (1976), p. 159-186

(a link to an online version can be found on the course webpage) and answer the following questions.

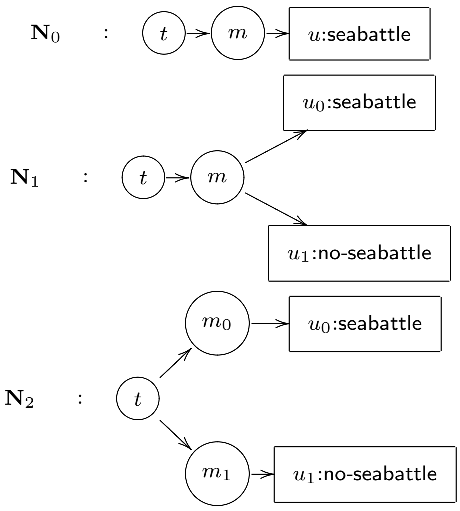
(1) Code pseudo-deduces the false statement (3) "Ford resigned last August" from the true statements (1) and (2). If Ford didn't resign, who did and when did he resign exactly? (1 point)
(2) Paraphrase Smullyan's solution to the problem of "The president resigned last August" in one sentence. (2 points)
(3) Does Code believe that Aristotle had something like Smullyan's solution in mind? (Give a brief argument; 2 points)
(4) Explain briefly (at most 100 words) what Code means when he says "Ford is not a spatio-temporal worm but rather ... a hydra". (3 points)

Exercise 13 (6 points).

A **naumachic model** is a quadruple $\langle M, U, \leq, S\rangle$ where M and U are finite sets, $\leq$ is a binary relation between M and U (*i.e.*, $\leq\ \subseteq M \times U$) and S is a function from U to $\{\mathsf{seabattle}, \mathsf{no\text{-}seabattle}\}$.
We call the elements of M **tomorrows**, the elements of U **day-after-tomorrows**, if $m \leq u$, we say that "u is a possible future of m", and if $S(u) = \mathsf{seabattle}$ we say that "there is a sea battle at u" (similarly, if $S(u) = \mathsf{no\text{-}seabattle}$ we say that "there is no sea battle at u").
Given a naumachic model $\mathbf{N} = \langle M, U, \leq, S\rangle$, we say

- $\mathbf{N} \models$ "There will be a sea battle the day after tomorrow" if for all $m \in M$ and all u such that $m \leq u$, $S(u) =$ seabattle.
- $\mathbf{N} \models$ "There will be no sea battle the day after tomorrow" if for all $m \in M$ and all u such that $m \leq u$, $S(u) =$ no-seabattle.
- $\mathbf{N} \models$ "Tomorrow it will be determined whether there is a sea battle the day after tomorrow" if for all $m \in M$ the following holds: all u such that $m \leq u$ have the same value of $S(u)$.

We consider the following four naumachic models (t represents "today", the m_i are the tomorrows, the u_i are the day-after-tomorrows, the arrows indicate the $\leq$ relation, and u_i:seabattle means $S(u_i) =$ seabattle).



Are the following statements true or false (1 point each)?

(1) In $\mathbf{N}_0$, there will be a sea battle the day after tomorrow.
(2) In $\mathbf{N}_1$, there will be a sea battle the day after tomorrow.
(3) In $\mathbf{N}_2$, there will be a sea battle the day after tomorrow.
(4) In $\mathbf{N}_0$, it will be determined tomorrow whether there is a sea battle the day after tomorrow.
(5) In $\mathbf{N}_1$, it will be determined tomorrow whether there is a sea battle the day after tomorrow.
(6) In $\mathbf{N}_2$, it will be determined tomorrow whether there is a sea battle the day after tomorrow.