

# Today...

- **Wrapping up the middle ages:**  
*obligationes*
- **1450-1650:**  
The infamous jump (“and after Ockham, there was Leibniz”).
- **Leibniz:**  
Arithmetization of reasoning (“*calculemus*”).

# Dialogic Logic (1).

*Paul Lorenzen* (1958): Explaining the meaning of propositional connectives via games and strategies.

- Two players, the **Proponent** and the **Opponent**.
- In the round 0, the Proponent has to assert the formula to be proved and the Opponent can make as many assertions as he wants. After that, the opponent starts the game.
- In all other moves, the players have to do an *announcement* and an *action*.
- An **announcement** is either of the form  $\text{attack}(n)$  or of the form  $\text{defend}(n)$ , interpreted as “I shall attack the assertion made in round  $n$ ” and “I shall defend myself against the attack made in round  $n$ ”.

# Dialogic Logic (2).

- An **action** can be one of the following moves:
  - assert( $\Phi$ ),
  - which one?,
  - left?,
  - right?,
  - what if?, assert( $\Phi$ ).
- You can only attack lines in which the other player asserted a formula. Depending on the formula, the following attacks are allowed:
  - $\Phi \vee \Psi$  may be attacked by which one?,
  - $\Phi \wedge \Psi$  may be attacked by left? or right?,
  - both  $\Phi \rightarrow \Psi$  and  $\neg\Phi$  may be attacked by “what if?, assert( $\Phi$ )”.

# Dialogic Logic (3).

- You can only defend against a line in which the other player attacked. Depending on the attack, the following defenses are allowed:
  - If  $\Phi \vee \Psi$  was attacked by which one?, you may defend with either  $\text{assert}(\Phi)$  or  $\text{assert}(\Psi)$ .
  - If  $\Phi \wedge \Psi$  was attacked by left?, you may defend with  $\text{assert}(\Phi)$ , if it was attacked by right?, you may defend with  $\text{assert}(\Psi)$ .
  - If  $\Phi \rightarrow \Psi$  was attacked by “what if?,  $\text{assert}(\Phi)$ ”, you may defend with  $\text{assert}(\Psi)$ .
  - You cannot defend an attack on  $\neg\Phi$ .

# Dialogic Logic (4).

The rules of the (constructive) game:

- In each move, the action and the announcement have to fit together, i.e., if the player announces  $\text{attack}(n)$  or  $\text{defend}(n)$ , then the action has to be an attack on move  $n$  or a defense against move  $n$ .
- In round  $n + 1$ , the Opponent has to either attack or defend against round  $n$ .
- An attack is called **open** if it has not yet been defended.
- The Proponent may attack any round, but may only defend against the most recent open attack. He may use any defense or attack against a given round at most once.
- The Opponent may assert any atomic formulas.
- The Proponent may assert only atomic formulas that have been asserted by the Opponent before.

# Dialogic logic (5).

If one player cannot make any legal moves anymore, the other player has won.

## Example 1.

0		—		$\text{assert}(p \wedge q \rightarrow q \wedge p)$
1	<b>attack</b> (0)	<b>what if?</b>	$\text{assert}(p \wedge q)$	
2				<b>attack</b> (1) <b>left?</b>
3	<b>defend</b> (2)		$\text{assert}(p)$	
4				<b>attack</b> (1) <b>right?</b>
5	<b>defend</b> (4)		$\text{assert}(q)$	
6				<b>defend</b> (1) $\text{assert}(q \wedge p)$
7	<b>attack</b> (6)		<b>left?</b>	
8				<b>defend</b> (7) $\text{assert}(q)$
9	—		—	

# Dialogic logic (5).

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## Example 1.

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1	<b>attack</b> (0)	<b>what if?</b>	$\text{assert}(p \wedge q)$	
2				<b>attack</b> (1) <b>left?</b>
3	<b>defend</b> (2)		$\text{assert}(p)$	
4				<b>attack</b> (1) <b>right?</b>
5	<b>defend</b> (4)		$\text{assert}(q)$	
6				<b>defend</b> (1) $\text{assert}(q \wedge p)$
7	<b>attack</b> (6)		<b>right?</b>	
8				<b>defend</b> (7) $\text{assert}(p)$
9	—		—	

# Dialogic logic (6).

We say that  $\Phi$  is **dialogically valid** if the Proponent has a winning strategy in the game in which he asserts  $\Phi$  in round 0.

In symbols:  $\models_{\text{dialog}} \Phi$ .

The dialogically valid formulas are exactly those provable in intuitionistic propositional logic.

## Example 2.

0		—	
1	attack(0)	what if? assert( $\neg\neg p$ )	assert( $\neg\neg p \rightarrow p$ )
2			attack(1)    what if? assert( $\neg p$ )
3	attack(2)	what if? assert( $p$ )	
4			—                                  —



# Dialogic Logic (7).

The rules of the (constructive) game:

- In each move, the action and the announcement have to fit together, i.e., if the player announces  $\text{attack}(n)$  or  $\text{defend}(n)$ , then the action has to be an attack on move  $n$  or a defense against move  $n$ .
- In round  $n + 1$ , the Opponent has to either attack or defend against round  $n$ .
- The Proponent may attack any round, but may only defend against the most recent open attack. He may use any defense or attack against a round at most once.
- The Opponent may assert any atomic formulas.
- The Proponent may assert only atomic formulas that have been asserted by the Opponent before.

# Dialogic Logic (7).

The rules of the (classical) game:

- In each move, the action and the announcement have to fit together, i.e., if the player announces  $\text{attack}(n)$  or  $\text{defend}(n)$ , then the action has to be an attack on move  $n$  or a defense against move  $n$ .
- In round  $n + 1$ , the Opponent has to either attack or defend against round  $n$ .
- The Proponent may attack and defend against any round. He may use any defense or attack against a round at most once.
- The Opponent may assert any atomic formulas.
- The Proponent may assert only atomic formulas that have been asserted by the Opponent before.

# Dialogic logic (8).

We say that  $\Phi$  is **classically valid** if the Proponent has a winning strategy in the (classical) game in which he asserts  $\Phi$  in round 0.

In symbols:  $\models_{\text{class}} \Phi$ .

The classically valid formulas are exactly those provable in classical propositional logic.

## Example 2a.

0	—	—	assert( $\neg\neg p \rightarrow p$ )
1	attack(0)	what if? assert( $\neg\neg p$ )	
2			attack(1) what if? assert( $\neg p$ )
3	attack(2)	what if? assert( $p$ )	
4			defend(1) assert( $p$ )
5	—	—	

# *Obligationes* (1).

*Obligationes*. A game-like disputation, somewhat similar to logic games. The origin is unclear, as is the purpose.

The name derives from the fact that one of the players is “obliged” to follow certain formal rules of discourse.

## **Different types of *obligationes*.**

- *positio*.
- *depositio*.
- *dubitatio*.
- *impositio*.
- *petitio*.
- *rei veritas / sit verum*.

# *Obligationes (2).*

- William of Shyreswood (1190-1249)
- Walter Burley (Burleigh; c.1275-1344)
- Roger Swyneshed (d.1365)
- Richard Kilvington (d.1361)
- William Ockham (c.1285-1347)
- Robert Fland (c.1350)
- Richard Lavenham (d.1399)
- Ralph Strode (d.1387)
- Peter of Candia
- Paul of Venice (c.1369-1429)

# *Obligationes (3).*

- **Walter Burley**, *De obligationibus*.  
Standard set of rules.
- **Roger Swyneshed**, *Obligationes* (1330-1335).  
Radical change in one of the rules results in a distinctly different system.

*responsio antiqua*

Walter Burley

William of Shyreswood

Ralph Strode

Peter of Candia

Paul of Venice

*responsio nova*

Roger Swyneshed

Robert Fland

Richard Lavenham

# *positio* according to Burley (1).

- Two players, the **opponent** and the **respondent**.
- The **opponent** starts by positing a *positum*  $\varphi^*$ .
- The **respondent** can “admit” or “deny”. If he denies, the game is over.
- If he admits the *positum*, the game starts. We set  $\Phi_0 := \{\varphi^*\}$ .
- In each round  $n$ , the **opponent** proposes a statement  $\varphi_n$  and the **respondent** either “concedes”, “denies” or “doubts” this statement according to certain rules. If the **respondent** concedes, then  $\Phi_{n+1} := \Phi_n \cup \{\varphi_n\}$ , if he denies, then  $\Phi_{n+1} := \Phi_n \cup \{\neg\varphi_n\}$ , and if he doubts, then  $\Phi_{n+1} := \Phi_n$ .

# *positio* according to Burley (2).

- We call  $\varphi_n$  **pertinent** (relevant) if either  $\Phi_n \vdash \varphi_n$  or  $\Phi_n \vdash \neg\varphi_n$ . In the first case, the **respondent** has to concede  $\varphi_n$ , in the second case, he has to deny  $\varphi_n$ .
- Otherwise, we call  $\varphi_n$  **impertinent** (irrelevant). In that case, the **respondent** has to concede it if he knows it is true, to deny it if he knows it is false, and to doubt it if he doesn't know.
- The **opponent** can end the game by saying *Tempus cedat*.



# Example 1.

## Opponent

I posit that Cicero was the teacher of Alexander the Great:  $\varphi^*$ .

Cicero was Roman:  $\varphi_0$ .

The teacher of Alexander the Great was Roman:  $\varphi_1$ .

## Respondent

I admit it.  $\Phi_0 = \{\varphi^*\}$ .

I concede it. Impertinent and true;  $\Phi_1 = \{\varphi^*, \varphi_0\}$ .

I concede it. Pertinent, follows from  $\Phi_1$ .

# Example 2.

## Opponent

I posit that Cicero was the teacher of Alexander the Great:  $\varphi^*$ .

The teacher of Alexander the Great was Greek:  $\varphi_0$ .

Cicero was Greek:  $\varphi_1$ .

## Respondent

I admit it.  $\Phi_0 = \{\varphi^*\}$ .

I concede it. Impertinent and true;  $\Phi_1 = \{\varphi^*, \varphi_0\}$ .

I concede it. Pertinent, follows from  $\Phi_1$ .

# Example 3 (“order matters!”)

## Opponent

I posit that Cicero was the teacher of Alexander the Great:  $\varphi^*$ .

The teacher of Alexander the Great was Roman:  $\varphi_0$ .

Cicero was Roman:  $\varphi_1$ .

## Respondent

I admit it.  $\Phi_0 = \{\varphi^*\}$ .

I deny it. Impertinent and false;  $\Phi_1 = \{\varphi^*, \neg\varphi_0\}$ .

I deny it. Pertinent, contradicts  $\Phi_1$ .

# Properties of Burley's *positio*.

- Provided that the *positum* is consistent, no disputation requires the **respondent** to concede  $\varphi$  at step  $n$  and  $\neg\varphi$  at step  $m$ .
- Provided that the *positum* is consistent,  $\Phi_i$  will always be a consistent set.
- It can be that the **respondent** has to give different answers to the same question (Example 4).
- The **opponent** can force the **respondent** to concede everything consistent (Example 5).

# Example 4.

Suppose that the **respondent** is a student, and does not know whether the King of France is currently running.

## Opponent

I posit that you are the Pope or the King of France is currently running:  $\varphi^*$

The King of France is currently running:  $\varphi_0$

You are the Pope:  $\varphi_1$ .

The King of France is currently running:  $\varphi_2 = \varphi_0$ .

## Respondent

I admit it.

$\Phi_0 = \{\varphi^*\}$ .

I doubt it.

Impertinent and unknown;  $\Phi_1 = \{\varphi^*\}$ .

I deny it.

Impertinent and false;  $\Phi_2 = \{\varphi^*, \neg\varphi_1\}$ .

I concede it.

Pertinent, follows from  $\Phi_2$ .

# *positio* according to Swyneshed.

- All of the rules of the game stay as in Burley's system, except for the definition of *pertinence*.
- In Swyneshed's system, a proposition  $\varphi_n$  is **pertinent** if it either follows from  $\varphi^*$  (then the **respondent** has to concede) or its negation follows from  $\varphi^*$  (then the **respondent** has to deny). Otherwise it is impertinent.

# Properties of Swyneshed's *positio*.

- Provided that the *positum* is consistent, no disputation requires the **respondent** to concede  $\varphi$  at step  $n$  and  $\neg\varphi$  at step  $m$ .
- The **respondent** never has to give different answers to the same question.
- $\Phi_i$  can be an inconsistent set (Example 6).

# Example 6.

Suppose that the respondent is a student in Paris, and not a bishop. Write  $\psi_0$  for “You are in Rome” and  $\psi_1$  for “You are a bishop”.

## Opponent

## Respondent

I posit that you are in Rome or you are a bishop:  $\psi_0 \vee \psi_1$

I admit it.

$\Phi_0 = \{\psi_0 \vee \psi_1\}$ .

You are in Rome or you are a bishop:  $\psi_0 \vee \psi_1$  .

I concede it.

Pertinent, follows from  $\Phi_0$ ;  $\Phi_1 = \{\psi_0 \vee \psi_1\}$ .

You are not in Rome:  $\neg\psi_0$  .

I concede it.

Impertinent, and true;  $\Phi_2 = \{\psi_0 \vee \psi_1, \neg\psi_0\}$ .

You are not a bishop:  $\neg\psi_1$  .

I concede it.

Impertinent, and true;  $\Phi_3 = \{\psi_0 \vee \psi_1, \neg\psi_0, \neg\psi_1\}$ .

$\Phi_2$  is an inconsistent set of sentences.



# *positio* according to Kilvington.

Richard Kilvington (d.1361).

- *Sophismata*, c.1325.
- *obligationes* as a solution method for *sophismata*.
- He follows Burley's rules, but changes the handling of impertinent sentences. If  $\varphi_n$  is impertinent, then the **respondent** has to concede if it **were true if the *positum* was the case**, and has to deny if it **were true if the *positum* was not the case**.

# *impositio*.

- In the *impositio*, the **opponent** doesn't posit a *positum* but instead gives a definition or redefinition.
- **Example 1.** “In this *impositio*, *asinus* will signify *homo*”.
- **Example 2.** “In this *impositio*, *deus* will signify *homo* in sentences that have to be denied or doubted and *deus* in sentences that have to be conceded.”

Suppose the **opponent** proposes “*deus est mortalis*”.

- If the **respondent** has to deny or doubt the sentence, then the sentence means *homo est mortalis*, but this is a true sentence, so it has to be conceded. Contradiction.
- If the **respondent** has to concede the sentence, then the sentence means *deus est mortalis*, but this is a false sentence, so it has to be denied. Contradiction.
- An *impositio* often takes the form of an insoluble.

# 1450–1550.

- **The Collapse of the Eastern Roman Empire 1453.**
- **The Discovery of the New World 1492.**
- **The Reformation 1517.**

# The Fall of Constantinople.



# The Fall of Constantinople.

**The Council of Ferrara-Florence, “the Union Council”**  
(1438-1445).

- **Georgius Gemistos Plethon**, (c.1355-1452).
- **Bessarion**, Bishop of Nicaea (c.1403-1472).
- **Johannes Argyropoulos** (1415-1487).

**May 29th, 1453.** Constantinople falls to the Ottomans;  
Constantine XI Palaeologos dies.

# The discovery of the Americas.



# The Reformation (1).



# The Reformation (1).

- Johannes Gutenberg (c.1400-1468).
- Alexander VI (Rodrigo Borgia; 1431-1503)
- Cesare Borgia (1475-1507; N. Machiavelli, *Il Principe*),  
Lucrezia Borgia (1480-1519)
- [The Peter Indulgence](#). Pope Julius II (1507).
- [Martin Luther](#) (1483 - 1546)
- Leo X (Giovanni de' Medici), Pope from 1513-1521.
- **October 31, 1517.** Luther posts the [95 Theses](#) on the church door.



# The Reformation (2).

- **1520.** Luther is excommunicated.
- **1527.** Philipp I the Magnanimous, *Landgraf* of Hesse (1504-1567) founds the first protestant university: **University of Marburg.**
- **1534.** The **Act of Supremacy**: Henry VIII becomes the head of the Church of England.
- **1542.** Pope Paul III founds the **Roman Inquisition** (= *Officium Sacrum*).
- **1571.** Pope Pius V founds the **Congregation of the Index.**

# Pierre de la Ramée.

**Pierre de la Ramée** (Petrus Ramus; 1515-1572)



- *Animadversiones in Dialecticam Aristotelis* (1543).
- Professor at the *Collège de France*.
- **Ramistic Logic**. *ars disserendi*. Logic of natural discourse.
- Protestant. Died in the **Massacre of St. Bartholomew** (August 24th, 1572).

# Port Royal.

- **Cornelius Jansen** (1585-1638), bishop of Ypres; *Augustinus* (1640), doctrine of strict predestination.
- **Abbey of Port Royal**, since 1638 centre of **Jansenism**.
- Pierre Nicole (1625-1695); Antoine Arnauld (1612-1694)
- **1662.** *La logique, ou l'art de penser*. Opposing scholasticism, “epistemological turn”.
- *Comprehension vs Extension*.
- Letters between Arnauld and **Leibniz**: 1687-1690.

# Comprehension vs Extension.

- *Comprehension of  $X$ .* The set of properties that  $x$  has to have in order to be an  $X$ .
- *Extension of  $X$ .* The set of all  $X$ .
- *An example.*
  - Universe of Discourse:  $U = \{a, A, A, B, b\}$
  - Properties: Consonant, Capital, Blue.
  - Extensions:
    - Consonant  $\rightsquigarrow \{B, b\}$
    - Capital  $\rightsquigarrow \{A, A, B\}$
    - Blue  $\rightsquigarrow \{a, B, b\}$
  - The Comprehension of Consonant in this universe of discourse includes the property blue.

# Leibniz (1).

**Gottfried Wilhelm von Leibniz** (1646-1716)



- Work on philosophy, mathematics, law (Doctorate in Law from the University of Altdorf (1667), alchemy, theology, physics, engineering, geology, history.
- Diplomatic tasks (1672).
- Attempts to build a calculating machine (1672).

# Leibniz (2).

- **1673-1677:** Invented **calculus** independently of **Sir Isaac Newton** (1643-1727).
- Research politics; foundation of Academies: Brandenburg, Dresden, Vienna, and St Petersburg.
- **1710:** *Théodicée*. “The best of all possible worlds”.

# Leibniz (3).

## Properties.

- *Identity of Indiscernibles*: If  $\{\Phi ; \Phi(x)\} = \{\Phi ; \Phi(y)\}$ , then  $x = y$ .
- Primary substances (“Plato”, “Socrates”) can be expressed in terms of properties: a uniform language of predication.
- Connected to Leibniz’ **monadology** (1714).

## Relations.

- Call for an analysis of relations.
- Attempt to reduce relations to unary predicates:  
“Plato is taller than Socrates”      **Taller**(Pla, Soc)  
“Plato is tall *in as much as* Socrates is short”      **Tall**(Pla)  $\oplus$  **Short**(Soc)

# *Calculemus!*

*“quando orientur controversiae, non magis disputatione opus erit inter duos philosophos, quam inter duos Computistas. Sufficiet enim calamos in manus sumere sedereque ad abacos, et sibi mutuo (accito si placet amico) dicere: **calculemus.**”*

~> **Arithmetization of Language** and **Automatization of Reasoning**



# Arithmetization of Language (1).

- *characteristica universalis*: general notation system for everything, based on the unanalyzable basics.
- *calculus ratiocinator*: formal system with a mechanizable deduction system.
- “*calculus de continentibus et contentis est species quaedam calculi de combinationibus*”
- The **properties** correspond to the natural numbers  $n > 1$ . The unanalyzable properties correspond to the prime numbers.
- **Example.** If *animal* corresponds to 2, and *rationalis* corresponds to 3, then *homo* would correspond to 6. If *philosophicus* corresponds to 5, then *philosophus* = *homo philosophicus* would be 30.

# Arithmetization of Language (2).

*animal*  $\rightsquigarrow$  2, *rationalis*  $\rightsquigarrow$  3, *homo*  $\rightsquigarrow$  6, *philosophicus*  $\rightsquigarrow$  5, *philosophus*  $\rightsquigarrow$  30.

- All individuals are determined by their properties, so *Socrates* is represented by a number  $n$ . Since *Socrates* is a philosopher,  $30|n$ .
- In general, “the individual represented by  $n$  has the property represented by  $m$ ” is rendered as  $m|n$ .
- Now we can formalize  $AaB$  and  $AiB$ . Let  $n_A$  and  $n_B$  be the numbers representing  $A$  and  $B$ , respectively.
  - $AaB$ :  $n_A|n_B$ .  
“Every human is an animal”:  $2|6$ .
  - $AiB$ :  $\exists k(n_A|k \cdot n_B)$ .  
“Some human is a philosopher”:  $30|5 \cdot 6$ .

# Arithmetization of Language (3).

$AaB: n_A|n_B; AiB: \exists k(n_A|k \cdot n_B).$

- **Barbara** becomes: “If  $n|m$  and  $m|k$ , then  $n|k$ .”  
So, the laws of arithmetic prove **Barbara**.
- **Darii** becomes: “If  $n|m$  and there is some  $w$  such that  $m|w \cdot k$ , then there is some  $w^*$  such that  $n|w^* \cdot k$ .”  
(Let  $w^* := w$ .)
- **But:**  $AiB$  is always true, as  $n|n \cdot m$  for all  $n$  and  $m$ .
- If  $n$  represents *homo* and  $m$  represents *asinus*, then  $n \cdot m$  would be a “man with the added property of being a donkey”.
- This simple calculus is not able to deal with negative propositions.