UNIVERSITEIT VAN AMSTERDAM
INSTITUTE FOR LOGIC, LANGUAGE AND COMPUTATION

Core Logic

2005/2006; 1st Semester

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Homework Set # 11 *Deadline:* November 29th, 2005

Exercise 36 (total of nine points).

Let $2^{\mathbb{N}}$ be the set of all infinite 0-1 sequences. For $x \in 2^{\mathbb{N}}$, we define $\widehat{x}(n) := 1 - x(n)$. We call $\mathcal{C} \subseteq 2^{\mathbb{N}}$ a **symmetric class** if for every $x \in \mathcal{C}$, we also have $\widehat{x} \in \mathcal{C}$. A function $F : \mathbb{N} \to \mathcal{C}$ is called a $\mathcal{C}$-good parametrization if the sequence $\langle F(n)(n) \,;\, n \in \mathbb{N}\rangle$ is an element of $\mathcal{C}$ and F is a surjection.

(1) Show that no symmetric class $\mathcal{C}$ can have a $\mathcal{C}$-good parametrization (4 points).
(2) Derive Cantor's Theorem ("there is no bijection between $\mathbb{N}$ and $2^{\mathbb{N}}$") as a corollary (2 points).
(3) Give an example of a (non-symmetric) class $\mathcal{C} \subseteq 2^{\mathbb{N}}$ that has a $\mathcal{C}$-good parametrization (3 points).

Exercise 37 (total of seven points).

Find three academic (PhD) students of **Alonzo Church** (1903-1995) (not the ones mentioned in the lecture, *i.e.*, Turing, Kleene, Davis, Rabin, Scott or Smullyan) and give:

- the student's name (½ point each),
- the title of the PhD thesis (½ point each),
- one theorem that the student proved (just the statement, no proof; 1 point each).

One of our guest lecturers is an academic great-grandchild of Church (*i.e.*, the PhD student of a PhD student of a PhD student of Church's). Who is it and what is his lineage (1 point)?

Exercise 38 (total of six points).

The ordinal ω_1^{CK} is sometimes called "the least admissible ordinal" and has an equivalent description in terms of an axiom system called "Kripke-Platek Set Theory" KP. Give a precise definition of ω_1^{CK} (2 points) and of KP (2 points) and give a brief (two to four sentences) description of the connection between the ordinal and KP (2 points).