UNIVERSITEIT VAN AMSTERDAM
INSTITUTE FOR LOGIC, LANGUAGE AND COMPUTATION

Core Logic

2004/2005; 1st Semester
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Homework Set # 7 *Deadline:* November 3rd, 2004

Exercise 18 (3 points total).

Give the names of the following medieval logicians and philosophers (1 point each):

- X was one of the Averroists of the XIIIth century, sometimes called "Radical Aristotelians". He was also one of the Modists. After the ban of Averroism in 1270, he retired to Scandinavia and died in Linköping.
- Y was one of the students of Anselm of Laon and taught a strongly realistic philosophy in Paris in the early XIIth century. After one of his students was very successful in argueing against Y's philosophy, Y retired to the abbey of St. Victor and was later made bishop of Châlons-sur-Marne.
- Z was an archbishop of Canterbury of Italian descent, immediate predecessor of Anselm of Canterbury. At the Council of Vercelli in 1050, he defended the doctrine of *transsubstantiation* against Berengar of Tours.

Exercise 19 (6 points total).

In this exercise, we consider the systems of *positio* as described by Walter Burley and Roger Swyneshed. If a *positum* φ^* is given and φ_k (for $0 \leq k \leq n$) are proposed sentences of the **Opponent**, we let Φ_k^{Burley} be the set of **"currently accepted truths"** according to Burley's system on the basis of the sequence $\langle \varphi^*, \varphi_0, \ldots, \varphi_n \rangle$.
Prove the following properties of the two systems:

(1) If the *positum* φ^* is consistent, then for all $k \leq n$, the set Φ_k^{Burley} is a consistent set (4 points).

(2) If the *positum* φ^* is consistent and $k < \ell \leq n$ with $\varphi_k = \varphi_\ell$, then the **Respondent** in a Swyneshed-style *positio* will give the same answer in steps k and ℓ of the *obligatio* (2 points).

Exercise 20 (6 points total).

We are considering a system reminiscent of Leibniz' attempts to arithmetize language. In the lecture, we introduced a system based on the divisor structure of the natural numbers, but this system was too simple as it didn't allow proper discussion of negative statements. Therefore, we add a number that should take care of the negative statements to the system. (The rough idea is: If 2 is *animal*, 3 is *rationalis* and 7 is *asinarius* ("donkey-like"), then $\langle 6, 7\rangle$ would represent *homo* (to preclude the option of constructing a *homo asinarius*) and $\langle 14, 3\rangle$ would represent *asinus* (to preclude the option of constructing an *asinus rationalis*.)

Formally: Call a pair $X := \langle \mathrm{p}_X, \mathrm{n}_X\rangle$ a **pseudo-Leibniz predicate** (**PLP**) if p_X and n_X are both positive natural numbers ≥ 2. We write $n|m$ for "n divides m" (*i.e.*, there is a $k \geq 1$ such that $nk = m$) and $n \perp m$ for "n and m are coprime" (*i.e.*, if $k|n$ and $k|m$, then $k = 1$). We define the following semantics for categorical propositions using PLP's:

$$
\begin{aligned}
X\mathrm{a}Y &:\equiv \mathrm{p}_X|\mathrm{p}_Y \;\&\; \mathrm{p}_Y \perp \mathrm{n}_X \\
X\mathrm{i}Y &:\equiv \exists k \geq 1(\mathrm{p}_X|k\cdot\mathrm{p}_Y \;\&\; k\cdot\mathrm{p}_Y \perp \mathrm{n}_X) \\
X\mathrm{e}Y &:\equiv \forall k \geq 1(\mathrm{p}_X|k\cdot\mathrm{p}_Y \;\vee\; \neg(k\cdot\mathrm{p}_Y \perp \mathrm{n}_X))
\end{aligned}
$$

In this semantics, **Barbara** can be expressed as:

$$\forall X, Y, Z\,((\mathrm{p}_X|\mathrm{p}_Y \;\&\; \mathrm{p}_Y|\mathrm{p}_Z \;\&\; \mathrm{p}_Y \perp \mathrm{n}_X \;\&\; \mathrm{p}_Z \perp \mathrm{n}_Y) \to \mathrm{p}_X|\mathrm{p}_Z \;\&\; \mathrm{p}_Z \perp \mathrm{n}_X)\,.$$

(1) Define a semantics for $X\mathrm{o}Y$ such that this is contradictory to $X\mathrm{a}Y$ (1 point).
(2) Give an example of a PLP that shows that **Barbara** is not valid with this semantics (2 points).
(3) Prove that **Celarent** is valid with this semantics (3 points).

Exercise 21 (10 points total).

Let $\mathbf{B} = \langle B, 0, 1, +, \cdot, -\rangle$ be a Boolean algebra. Define an operation $\star$ by $x \star y := -(x + y)$ (the `NOR` or Sheffer operation).

(1) Give formulas $\varphi_{\mathrm{mult}}, \varphi_{\mathrm{add}}, \varphi_{\mathrm{comp}}$ in the language just containing $\star$, $=$ and parentheses such that

$$
\begin{aligned}
\varphi_{\mathrm{mult}}(x, y, z) &\equiv x \cdot y = z \\
\varphi_{\mathrm{add}}(x, y, z) &\equiv x + y = z \\
\varphi_{\mathrm{comp}}(x, z) &\equiv -x = z
\end{aligned}
$$

(1 point each). (In other words, the $\star$-language is expressive enough to define the language of Boolean algebras.)

(2) Prove that the following three so-called "Sheffer axioms" hold for $\star$:

$$(x \star x) \star (x \star x) = x \text{ (1 point)}$$

$$x \star (y \star (y \star y)) = x \star x \text{ (1 point)}$$

$$(x \star (y \star z)) \star (x \star (y \star z)) = ((y \star y) \star x) \star ((z \star z) \star x) \text{ (1½ points).}$$

(3) Prove that there cannot be any three-element Boolean algebra (3½ points).