UNIVERSITEIT VAN AMSTERDAM
INSTITUTE FOR LOGIC, LANGUAGE AND COMPUTATION

Caput Logic, Language and Information

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Homework Set # 6 *Deadline:* October 26th, 2004

Please submit the solutions to the mailbox "S. Bold" on ground floor in Euclides, or by e-mail to `sbold@science.uva.nl` **before 17:00 on October 26th, 2004.**

Exercise 1 (½ Point). Let $\mathbf{G} = \langle V_{\mathbf{G}}, E_{\mathbf{G}} \rangle$ be a graph and $\mathbf{T}_{\mathbf{G}}$ its unraveled tree. Prove that a finite graph $\mathbf{G}$ contains cycles if and only if its unraveled tree $\mathbf{T}_{\mathbf{G}}$ is infinite.

Exercise 2 (1 Point).

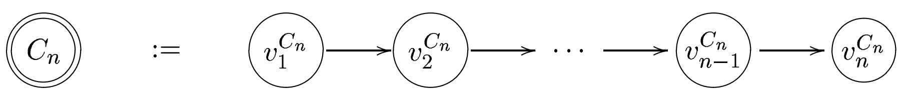
Consider the game of **Nim** with the usual rules. Write $\mathrm{Nim}(n_0, \ldots, n_k)$ for the Nim game with $k+1$ heaps where the ith heap has size n_i.

(1) Consider the games $\mathrm{Nim}(1, n, m)$. Give conditions on n and m such that the starting player wins the game. Give an argument for the correctness of your conditions.

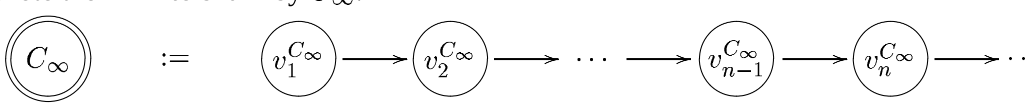
(2) Does the starting player win $\mathrm{Nim}(17, 5, 3)$? Give an argument for your answer.

Exercise 3 (1½ Points).

Let $I = \{0, 1\}$ be the set of players. We call the following graph a **chain of length** n:



We denote the **infinite chain** by C_∞:



In all of these graphs C, we let $\mu(v_i^C) = 1$ if i is odd and $\mu(v_i^C) = 0$ if i is even. We let $P(v_n^{C_n}) = 0$ if n is odd and $P(v_n^{C_n}) = 1$ if n is even.

Consider now the following game with draw. Take the game tree $\mathbf{T}$ defined in Figure 1. The moving function μ is defined as follows:

- On elements of C_n or C_∞, μ is defined as above.
- $\mu(s) = \mu(v) = 0$.
- $\mu(w) = 1$.

The payoff function P is defined as follows:

- If W is a finite walk, then it ends in some $v_n^{C_n}$. In that case, let $P(W) := P(v_n^{C_n})$ as defined above.
- If W is an infinite walk, then $P(W)$ is a draw.

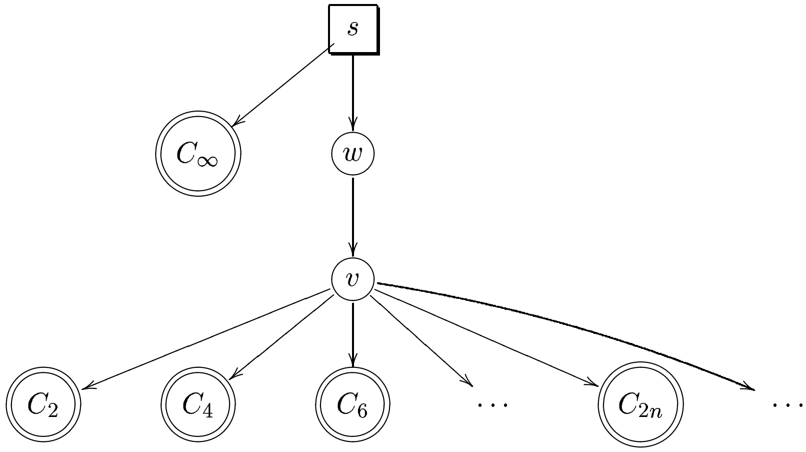


FIGURE 1

(1) Argue that the fixed point of the backwards induction procedure is a transfinite ordinal.
(2) Argue that the labelling at the fixed point is not a total function.
(3) Argue that the analysis of $\mathbb{G}(\mathbf{T}, \mu, P)$ needs the **totalization** step.