M-Theory on Calabi-Yau 5-folds

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Work with Alexander Haupt and André Lukas;

previous work with Hong Lü, Chris Pope and Paul Townsend

Motivation

- CY_3 manifolds provide one of the most important approaches to phenomenological contact between realistic physics and string/M-theory.
- The standard embedding of an SU(3) spin connection into the heterotic string's $E_8 \times E_8$ gauge group breaks the YM gauge group down to $E_8 \times E_6$ and E_6 is physically appealing.
- At the same time, from an M-theory perspective, the 4+7 split is unnatural. A more "democratic" formulation of the spatial dimensions would seem more natural.
- Cosmology could naturally involve a 1+10 split. All space dimensions would initially be treated as compact, in anticipation of 3 of them expanding.

Overview

- Review of bosonic sector of D=11 supergravity including normalizations Bilal
- Topological considerations and flux quantization in M-theory
  - topological constraint on compact 10-manifolds
- CY moduli sigma model
- 2-component local supersymmetry in D=1
- Effect of α' corrections on CY_5 geometrical structure
- Supersymmetry preservation and generalized holonomy

D=11 supergravity

$$I_{11} = I_{CJS,B} + I_{CJS,F} + I_{GS} + \dots .$$

$$I_{CJS,B} = \frac{1}{2\kappa_{11}^2} \int_{\mathcal{M}} \left\{ R * 1 - \frac{1}{2} G \wedge *G - \frac{1}{6} G \wedge G \wedge C \right\}$$

$$G_{[4]} = dC_{[3]}$$

4-form field strength for 3-form gauge field

$$I_{CJS,F} = -\frac{1}{2\kappa_{11}^2} \int_{\mathcal{M}} d^{11}x \sqrt{-g} \Big\{ \bar{\psi}_M \Gamma^{MNP} D_N(\omega) \psi_P + \frac{1}{96} \left(\bar{\psi}_M \Gamma^{MNPQRS} \psi_S + 12 \bar{\psi}^N \Gamma^{PQ} \psi^R \right) G_{NPQR} + (\mathrm{fermi})^4 \Big\}$$

- The above terms combine to form an invariant under the classical supersymmetry transformations

$$\delta_\epsilon g_{MN} = 2\bar{\epsilon} \Gamma_{(M} \psi_{N)},$$

$$\delta_\epsilon C_{MNP} = -3\bar{\epsilon} \Gamma_{[MN} \psi_{P]},$$

$$\delta_\epsilon \psi_M = 2 D_M(\omega) \epsilon + \frac{1}{144} (\Gamma_M{}^{NPQR} - 8 \delta_M^N \Gamma^{PQR}) \epsilon G_{NPQR} + (\mathrm{fermi})^3 .$$

- Variation of the Cremmer-Julia-Scherk action leads to the classical supergravity field equations:

$$R_{MN} = \frac{1}{12} G_{MM_2 \ldots M_4} G_N{}^{M_2 \ldots M_4} - \frac{1}{144} g_{MN} G_{M_1 \ldots M_4} G^{M_1 \ldots M_4}$$

$$d * G + \frac{1}{2} G \wedge G = 0.$$

$$\Gamma^{MNP} D_N(\omega) \psi_P + \frac{1}{96} \left(\Gamma^{MNPQRS} \psi_S + 12 \delta^{MN} \Gamma^{PQ} \psi^R \right) G_{NPQR} + (\text{fermi})^3 = 0$$

- Quantum corrections change these equations in a way that is important for CY_5 compactifications. Among the $\beta = (2\pi)^2 \alpha'^3$ quantum corrections is a Green-Schwarz type term needed for M_5-brane worldvolume anomaly cancellations. Vafa & Witten
Duff, Liu & Minasian

- This GS term is a superpartner of the $R^4_{\mu\nu\rho\sigma}$ effective action corrections.

- The classical CJS equation for $C_{[3]}$

$$d*G+\frac{1}{2}G\wedge G=0$$

is accordingly modified by the Green-Schwarz correction

$$I_{GS}=\frac{-(2\pi)^4\beta}{2\kappa_{11}^2}\int C\wedge X_8$$

where $X_8=\frac{1}{(2\pi)^4}\left[-\frac{1}{768}(trR^2)^2+\frac{1}{192}trR^4\right]$

- This gives rise to the quantum-corrected equation

$$d*G+\frac{1}{2}G\wedge G+(2\pi)^4\beta X_8=0.$$

- The Green-Schwarz correction term is necessary for cancelation of anomalies on the d=6 worldvolumes of 5-branes: $\beta = \dfrac{1}{(2\pi)^3 T_5}$ T_5 = 5-brane tension

- One also has the Dirac quantization condition
$$T_2 T_5 = \frac{2\pi}{2\kappa_{11}^2}$$
T_2 = 2-brane tension

and the condition
$$T_5 = \frac{1}{2\pi} T_2^2$$
de Alwis

which is needed, e.g., for invariance under large 3-form gauge transformations.

Lavrinenko, Lü, Pope & K.S.S

Kalkkinen & K.S.S

- Putting these together, have
$$T_2 = \left(\frac{2\pi^2}{\kappa_{11}^2}\right)^{1/3} \quad \beta = \left(\frac{2\kappa_{11}^2}{(2\pi)^5}\right)^{2/3} \quad 2\kappa_{11}^2 = (2\pi)^8 (\alpha')^{9/2}$$

Topological considerations

A. Haupt, A. Lukas & K.S.S.

- Corrected 3-form field equation:

$$d * G + \frac{1}{2} G \wedge G + (2\pi)^4 \beta X_8 = 0$$

where

$$X_8 = \frac{1}{48}\left(\left(\frac{p_1}{2}\right)^2 - p_2\right)$$

$$p_1 = -\frac{1}{2}\left(\frac{1}{2\pi}\right)^2 \mathrm{tr} R^2$$

$$p_2 = \frac{1}{8}\left(\frac{1}{2\pi}\right)^4 \left((\mathrm{tr} R^2)^2 - 2\mathrm{tr} R^4\right)$$

1st & 2nd Pontriagin classes

- Now specialize to $M_{11} = \mathbb{R} \times CY_5$ and simplify above relations:

$$p(T(\mathbb{R} \times CY_5)) = p(T(\mathbb{R})) \wedge p(T(CY_5))$$

$p(T(\mathbb{R})) = 1$ so $p(T(M_{10}))$ is given by $p(T(CY_5))$

- Now, for complex manifolds, there are relations between Pontriagin and Chern classes:

$$p_1 = c_1^2 - 2c_2$$
$$p_2 = 2c_4 - 2c_1c_3 + c_2^2$$

T. Hübsch

so for the case of a Calabi-Yau manifold with $c_1 = 0$ one has $\left(\frac{p_1}{2}\right)^2 - p_2 = -2c_4$ and consequently $X_8 = -\frac{1}{24}c_4$

- Define $g = \frac{1}{(2\pi)^2\beta^{1/2}}G$ and use the corrected field equations together with the fact that $d * G$ is exact to deduce $\left[\frac{1}{2}G \wedge G + (2\pi)^4 \beta X_8\right] = 0$ giving the <u>topological constraint</u>

$$c_4(CY_5) - 12[g] \wedge [g] = 0$$

4-form flux quantization

- 2-branes couple to the $C_{[3]}$ background via

$$S_{WZ}^{2br} = T_2 \int_{W_3} C \to T_2 \int_{D_4} G \qquad \partial D_4 = W_3$$

- This gives the flux quantization condition

$$[g] - \frac{p_1}{4} \in H^4(CY_5, \mathbb{Z})$$

Witten

or, for $c_1 = 0$,

$$[g] + \frac{c_2}{2} \in H^4(CY_5, \mathbb{Z}) \qquad g = \frac{T_2}{2\pi} G$$

- Thus, depending on the value of the 2nd Chern class c_2 , the normalized flux g is quantized in integer or half-integer units.

- Happily, this is consistent with the topological constraint $c_4(CY_5) - 12[g] \wedge [g] = 0$

- For complete intersection compact CY_5, analysis shows that $c_4\left(CY_5^{c.i.}\right) > 0$ requiring $[g] \neq 0$ so 4-form flux must be turned on at order $\sqrt{\beta}$

- However, one can make orbifold constructions with $c_4 = 0$.

- Non-compact CY_5 can also have $c_4 = 0$.

- In cases with $c_4 = 0$, the flux is turned on at order β

CY_5 moduli $D=1$ Supersymmetric sigma model

- CY_5 Hodge diamond:

$$
\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 0 & & 0 & & & & \\
 & & & 0 & & h^{1,1} & & 0 & & & \\
 & & 0 & & h^{1,2} & & h^{1,2} & & 0 & & \\
 & 0 & & h^{1,3} & & h^{2,2} & & h^{1,3} & & 0 & \\
1 & & h^{1,4} & & h^{2,3} & & h^{2,3} & & h^{1,4} & & 1
\end{array}
$$

- Hirzebruch-Riemann-Roch theorem with $c_1 = 0$:

$$11h^{1,1} - 10h^{1,2} - h^{2,2} + h^{2,3} + 10h^{1,3} - 11h^{1,4} = 0$$

so there are 6-1=5 independent Hodge numbers. The corresponding harmonic forms contribute $D=1$ massless Kaluza-Klein modes.

- Metric:

$$ds^2 = -Nd\tau^2 + 2g_{rs}(x, \varphi^I(\tau))dx^r dx^s$$

$$\varphi^I(\tau) = (t^i(\tau), z^a(\tau), z^{\bar{a}}(\tau))$$

$h^{1,1}$ $h^{1,4}$ moduli

in complex coordinates

$$x^r \longrightarrow x^\mu,\ x^{\bar{\nu}} \qquad \mu,\ \bar{\nu} = 1, \ldots, 5$$

$$\delta g_{\mu\bar{\nu}} = \delta t^i \omega_{i\mu\bar{\nu}} \quad \delta g_{\mu\nu} = \delta z^{\bar{a}} b_{\bar{a}\mu\nu} \quad \delta g_{\bar{\mu}\bar{\nu}} = \delta z^a b_{a\bar{\mu}\bar{\nu}}$$

$$b_{\bar{a}\mu\nu} = \frac{\mathrm{i}}{||\Omega||^2} \Omega_\mu{}^{\bar{o}\bar{\rho}\bar{\sigma}\bar{\tau}} \chi_{\bar{a}\bar{o}\bar{\rho}\bar{\sigma}\bar{\tau}\nu}$$

(5,0) volume form (4,1) harmonic form

$\omega_i \in \mathrm{Harm}(1,1)$

$\chi_a \in \mathrm{Harm}(1,4)$

- 3-form field:

$$\delta C = \xi^p(\tau)\nu_p + \text{c.c.}$$

$h^{1,2}$

$\nu_p \in \mathrm{Harm}(1,2)$

Fermionic zero modes

- Expand $\Psi_M(\tau, x^r)$ using the Killing spinor $\eta(x^r)$ on CY$_5$, e.g. $\Psi_0(\tau, x^r) = \bar{\psi}_0(\tau)\eta(x^r) + \mathrm{cc}$ $\qquad \eta^\dagger\eta = 1$
- For $\Psi_\mu(x^r)$, $\Psi_{\bar{\nu}}(x^r)$ the expansion uses the (1,1), (2,1), (3,1) and (4,1) harmonic forms:

$$\psi_{\bar{\mu}} = \psi^i(\tau) \otimes (\underbrace{\omega_{i\alpha_1\bar{\mu}}}_{(1,1)}\gamma^{\alpha_1}\eta) + \frac{1}{4}\lambda^p(\tau) \otimes (\underbrace{\nu_{p\alpha_1\alpha_2\bar{\mu}}}_{(2,1)}\gamma^{\alpha_1\alpha_2}\eta)$$

$$+ \frac{1}{4!}\rho^x(\tau) \otimes (\underbrace{\varpi_{x\alpha_1\ldots\alpha_3\bar{\mu}}}_{(3,1)}\gamma^{\alpha_1\ldots\alpha_3}\eta) - \frac{1}{4!}\kappa^a(\tau) \otimes (||\Omega||^{-1}\underbrace{\chi_{a\alpha_1\ldots\alpha_4\bar{\mu}}}_{(4,1)}\gamma^{\alpha_1\ldots\alpha_4}\eta)$$

$$\psi_\mu = (\psi_{\bar{\mu}})^*$$

- The (3,1) species has no bosonic partners, however. This points out a strange feature of supersymmetric life in $D = 1$: on-shell bosonic and fermionic degrees of freedom do not have to balance. Coles & Papadopoulos

- What happens to the other possible types of harmonic forms, *e.g.* (3,2), (2,2) and (5,0)?
    - These are reabsorbed into the (1,1) and (2,1) harmonic types.
    - To see this, one needs to use the $\gamma^{\bar{\mu}}\eta = 0$ property of CY Killing spinors together with the Dirac algebra $\{\gamma^{\mu}, \gamma^{\bar{\nu}}\} = 2g^{\mu\bar{\nu}}$ and Fierz identities to reduce these species to other types. *E.g.* the (5,0) type is converted into a (1,1) species, and is the superpartner of the CY volume modulus.

Bosonic sigma model

gauge N=1

$$I^{\mathrm{B}}_{\mathrm{CJS}} \underset{M_{11}=\mathbb{R}\times CY_5}{\longrightarrow} \int d\tau \left\{ \frac{1}{4} G^{(1,1)}_{ij}(t)\dot{t}^i\dot{t}^j + G^{(2,1)}_{p\bar{q}}(t)\dot{\xi}^p\dot{\bar{\xi}}^{\bar{q}} - 4V(t)G^{(4,1)}_{a\bar{b}}(z,\bar{z})\dot{z}^a\dot{\bar{z}}^{\bar{b}} \right\}$$

$$G^{(1,1)}_{ij} = \partial_i\partial_j K^{(1,1)} - 25\frac{K_iK_j}{K^2} \qquad K^{(1,1)} = -\frac{1}{2}\ln K$$

$$K = \int J\wedge J\wedge J\wedge J\wedge J \qquad J = t^i\omega_i \quad \text{complex structure}$$

$$= d_{i_1\ldots i_5}t^{i_1}\ldots t^{i_5} \qquad d_{i_1\ldots i_5}\text{: intersection numbers}$$

$$K_i = \int \omega_i\wedge J\wedge J\wedge J\wedge J = d_{ij_1\ldots j_4}t^{j_1\cdots j_4}$$

$$G^{(4,1)}_{a\bar{b}} = \partial_a\partial_{\bar{b}}K^{(4,1)} \qquad K^{(4,1)} = -\ln(\mathrm{i}(G_{\bar{a}}z^{\bar{a}} - z^a\bar{G}_a))$$

$$G^{(2,1)}_{p\bar{q}} = -2\int_X \nu_p\wedge {}^*\bar{\nu}_{\bar{q}} \; = \; \mathrm{i}d_{p\bar{q}ij}t^it^j \qquad \text{Canonical inner product}$$

- Notes
  - The (1,1) metric is not a canonical special Kähler metric but it is determined by intersection numbers (topological data), as is the canonical (2,1) metric.
  - The (4,1) metric is the canonical Weil-Peterson metric (very special Kähler) but it is determined by a prepotential (involving non-topological data).
  - The Kähler and complex structure sectors don't decouple owing to the $V(t)$ factor.

D=1 supersymmetry multiplets

- Inserting the D=11 supersymmetry transformations into the reduction ansatz, one finds the surviving 2-component D=1 supersymmetry (CY_5 breaks supersymmetry to 1/16).
- One finds two kinds of D=1 supermultiplets
  - (2a) real $\phi = \bar{\phi}$ $\quad \phi = \varphi + i\theta\psi + i\bar{\theta}\bar{\psi} - \frac{1}{2}\theta\bar{\theta}f$
  - (2b) *i.e.* (2,0) chiral $\bar{D}Z = 0 \quad Z = z + \theta\kappa - \frac{i}{2}\theta\bar{\theta}\dot{z}$
- Local D=1 supersymmety is described by the supervielbeins E_M^A , $\nabla_A = E_A^M \partial_M$, $[\nabla_A, \nabla_B\} = -T_{AB}^C \nabla_C$

subject to the torsion constraints

No D=1 curvature!

$T_{\underline{\theta\bar{\theta}}}{}^0 = i \;\; (0)$, $\quad T_{\underline{\theta\bar{\theta}}}{}^\theta = 0 \;\; (\frac{1}{2})$ "conventional"

$T_{\underline{\bar{\theta}\bar{\theta}}}{}^0 = 0 \;\; (0)$, $\quad T_{\underline{\bar{\theta}\bar{\theta}}}{}^\theta = 0 \;\; (\frac{1}{2})$ "representation preserving"

$T_{\underline{\theta\theta}}{}^\theta = 0 \;\; (\frac{1}{2})$ "type 3"

- D=1 supergravity plays an entirely destructive rôle: it's effect is merely to impose constraints on the D=1 supermatter that couples to it. Subject to the torsion constraints, the remaining supergravity fields are the einbein and D=1 gravitino, contained in

$$\mathcal{E} := \mathrm{sdet} E_A{}^B = N - \frac{i}{2}\theta\bar{\psi}_0 - \frac{i}{2}\bar{\theta}\psi_0$$

  - Consider for example a supergravity coupled (2b) action for a single multiplet $S = \int d\tau d^2\theta \mathcal{E} \nabla Z \bar{\nabla} \bar{Z} = \int d\tau \mathcal{L}$. In component fields, this Lagrangian is

$$\mathcal{L} = N^{-1}\dot{Z}\dot{\bar{Z}} - \frac{i}{2}(\kappa\dot{\bar{\kappa}} - \dot{\kappa}\bar{\kappa}) - N^{-1}(\psi_0\kappa\dot{\bar{Z}} + \bar{\psi}_0\bar{\kappa}\dot{Z}) - N^{-1}\psi_0\bar{\psi}_0\kappa\bar{\kappa}$$

    and varying with respect to N and ψ_0 one finds

$$Z = (\text{const.}) \qquad\qquad \kappa = 0$$

- In the full supergravity-coupled action, the constraints link the (2a) and (2b) sectors.

- The full D=1 supergravity-coupled action is

$$I_1 = I_{11}\Big|_{\mathbb{R}\times X} = -\frac{m}{2}\int d\tau d^2\theta\mathcal{E}\left\{G^{(1,1)}_{ij}(T)\nabla T^i\bar{\nabla}T^j + G^{(2,1)}_{p\bar{q}}(T)\nabla\Xi^p\bar{\nabla}\bar{\Xi}^{\bar{q}}\right.$$
$$\left. + G^{(3,1)}_{x\bar{y}}(T)\hat{\mathcal{R}}^x\bar{\hat{\mathcal{R}}}^{\bar{y}} + 4\mathcal{V}(T)G^{(4,1)}_{a\bar{b}}(Z,\bar{Z})\bar{\nabla}\mathcal{Z}^a\nabla\bar{\mathcal{Z}}^{\bar{b}}\right\}$$

- Agreement between this superspace action and the Kaluza-Klein dimensionally reduced action has been checked through $(\text{fermi})^2$ terms. The leading bosonic terms reproduce the component action given above.

- *After* varying the action to obtain the supergravity constraints, one can make the gauge choices

$$N = 1,\ \psi_0 = 0$$

Quantum $\beta \leftrightarrow \alpha'^3$ corrections

Lü, Pope, Townsend, K.S.S

- The $\beta \int C_{[3]} \wedge X_8$ term is a D=11 superpartner to other bosonic corrections including R^4_{ABCD} terms.
- Specialize to the topologically simplest case where $c_4 = 0$ - either noncompact CY_5 or an orbifold construction.
- Correction terms of relevance:

Gross & Witten; Peeters, Vanhove & Westerberg

$$\Delta L = \frac{\beta}{1152}(Y + 2Y_2 + \ldots)^* + (2\pi)^4 \beta C \wedge X_8$$

plus terms that vanish for $R_{MN} = 0$

$$Y_{\text{string light cone}} \sim \int d^{16}\psi \exp\left[(\bar{\psi}_- \Gamma^{ij} \psi_-)(\bar{\psi}_+ \Gamma^{kl} \psi_+) R_{ijkl}\right]$$

Indices extended to 11 values

- D=11 extension of type IIA string correction
- Berezin integral $\rightarrow R^4$ terms only

- Varying Y, get for initially Ricci-flat spaces

$$\delta \int \sqrt{-g} Y \, d^{11}x = \int \sqrt{-g} \left(X_{rs} + \nabla_r \nabla_s Z - g_{rs} \Box Z \right) \delta g^{rs}$$

$$X_{rs} = \nabla^t \nabla^u X_{rstu}$$

X_{rstu} cubic in curvatures

$$Z = R_{ijkl} R^{klmn} R_{mn}{}^{ij} - 2 R_{ikjl} R^{kmln} R_m{}^i{}_n{}^j$$

- The Y_2 correction term is of Lovelock form:

$$Y_2 = \frac{315}{2} R^{[m_1 m_2}{}_{m_1 m_2} \cdots R^{m_7 m_8]}{}_{m_7 m_8}$$

Lift to D=11 of D=8 Euler integrand

- Varying Y_2, get

$$\delta \int \sqrt{-g} Y_2 = \int \sqrt{-g} E_{mn} \delta g^{mn}$$

Lovelock
Deruelle

$$E_m{}^n = -\frac{9!}{2^9} \delta^{n n_1 \cdots n_8}_{m m_1 \cdots m_8} R^{m_1 m_2}{}_{n_1 n_2} \cdots R^{m_7 m_8}{}_{n_7 n_8}$$

- Consequently, the corrected field equations are

$$\hat{R}_{00} - \frac{1}{2} g_{00} \hat{R} = -\frac{\beta}{1152} \Box Z g_{00} + \frac{\beta}{576} E_{00}$$

$$\hat{R}ij - \frac{1}{2} g_{ij} \hat{R} = \frac{\beta}{1152} (X_{ij} + \nabla_i \nabla_j Z - g_{ij} Z) + \frac{\beta}{576} E_{ij}$$

$\hat{R}_{mn}$: D=11 Ricci

- To solve these, we need to introduce a *warp factor* in the metric:

$$ds_{11}^2 = -e^{2A(x^r)} d\tau^2 + e^{-\frac{1}{4}A(x^r)} ds_{10}^2$$

  - then for the Ricci tensor one has

$$\hat{R}_{00} = \Box A \qquad \hat{R}_{ij} = R_{ij} + \frac{1}{8} g_{ij} \Box A$$

Rij: D=10 Ricci

$\Box = \nabla^2$

so $\hat{R} = R + \frac{1}{4} \Box A$ and hence

$$R_{ij} = \frac{\beta}{1152} \left(X_{ij} + \nabla_i \nabla_j Z + 2E_{ij} - \frac{1}{4} E_k{}^k g_{ij} \right)$$

- For an initially Kähler manifold, one finds

$$X_{ij} = \nabla_{\hat{i}} \nabla_{\hat{j}} Z = J_i{}^k J_j{}^l \nabla_k \nabla_l Z$$

$J_i{}^j$: complex structure

and $E_k{}^k = -Y_2$

so
$$\Box A = \frac{\beta}{1728} Y_2$$

$$R_{ij} = \frac{\beta}{1152} \left(\nabla_{\hat{i}} \nabla_{\hat{j}} Z + \nabla_i \nabla_j Z + 2E_{ij} + \frac{1}{4} Y_2 g_{ij} \right)$$

terms expected from CY_3 case

terms arising from Y_2

- These corrections have the effect of making the Ricci tensor non-vanishing, and even remove the Kähler property of the metric. Nontheless, the manifold remains special, as we shall see.

Gravitational sourcing of 4-form flux

- The corrected 3-form field equation is

$$d * G + \frac{1}{2} G \wedge G + (2\pi)^4 \beta X_8 = 0$$

    - for initial purely gravitational backgrounds with $c_4 = 0$, this forces 4-form flux to turn on at order β

- Let $\hat{G}_{[4]} = G_{[3]} \wedge d\tau + G_{[4]}$
    - for $c_4 = 0$, assume $G_{[4]} = 0$
    - then $d * G_{[3]} = (2\pi)^4 \beta X_8$ D=10 Hodge dual here
    - in turn, write $G_{[3]} = \frac{3}{4} J \wedge dA + \tilde{G}_{[3]}$ $\quad J^{jk} \tilde{G}_{ijk} = 0$

- Then the Einstein equation becomes

$$R_{ij} = \frac{3}{8}(\nabla_i \nabla_j A + \nabla_{\hat{i}} \nabla_{\hat{j}} A) + \frac{\beta}{1152}(\nabla_i \nabla_j Z + \nabla_{\hat{i}} \nabla_{\hat{j}} Z) - \frac{1}{2} \nabla^k \tilde{G}_{i\hat{j}k}$$

- The gravitational sourcing of 4-form flux is accompanied by changes to the Killing spinor and to the complex structure.

- Killing spinor equation: $\hat{D}_m \eta = 0$ becomes deformed, requiring a brane-like warp factor $\hat{\eta} = e^{\frac{1}{2}A}\eta$ and

$$D_i \eta = \nabla_i \eta + \mathrm{i}(\nabla_{\hat{i}} h)\eta + \frac{\mathrm{i}}{8}\tilde{G}_{ijk}\gamma^{jk}\eta = 0$$

$$h = \frac{3}{16}A + \frac{\beta}{2304}Z \qquad \gamma_{11}\eta = -\eta \qquad \tilde{G}_{ijk}\gamma^{ijk}\eta = 0$$

- The deformed Killing spinor leads to a deformed complex structure $J_{ij} = -\mathrm{i}\bar{\eta}\gamma_{ij}\eta$

$$\nabla_j J_i{}^j = \frac{1}{2}\tilde{G}_{ij}{}^k - \frac{1}{2}\tilde{G}_{\hat{i}\hat{j}}{}^{\hat{k}} \neq 0$$

so the deformed space <u>is no longer Kähler</u>

- Despite the loss of Kähler structure, the Nijenhuis tensor $N_{ij}{}^{k} = \partial_{[i}J_{i]}{}^{k} - J_{i}{}^{l}J_{j}{}^{m}\partial_{[m}J_{l]}{}^{k}$ still vanishes, so the deformed space is still a complex manifold.

- It no longer has $SU(5)$ holonomy, but one may still define a generalized holonomy for the Killing spinor operator D_i. The generalized transverse structure group is $SL(16,\mathbb{C})$. The decomposition of the deformed Killing spinor under the generalized holonomy still contains singlets, showing that supersymmetry remains unbroken.

Deformed special holonomy

- The effect of the α'^3 corrections is to destroy the original special holonomy, giving a general complex D=10 manifold.
- Nonetheless, the specific structure of the α'^3 corrections is such as to *permit* the corrected Einstein equation to arise as the integrability condition for an α'^3 corrected Killing spinor equation.
- This fits into a general pattern that obtains also for 7-manifolds of initially G_2 and 8-manifolds of initially $Spin_7$ holonomies.

- In all cases (including the D≤8 Kähler cases where the effect of the corrections is simply to include an extra *U(1)* factor in the holonomy), supersymmetry can be preserved providing the Killing spinor equation acquires its own α'^3 correction, *e.g.* for the D≤8 cases

$$D_i\eta = (\nabla_i + \xi(\alpha')^3 Q_i)\eta = 0$$

where

Candelas, Freeman, Pope, Sohnius & K.S.S

$$Q_i = -\tfrac{3}{4}(\nabla^j R_{ikm_1m_2})\, R_{j\ell m_3m_4}\, R^{k\ell}{}_{m_5m_6}\, \Gamma^{m_1\cdots m_6}$$

- In the various cases of initially special holonomy, this can be rewritten in ways that more directly yield the corrected Einstein equation as integrability conditon, *e.g.* in the G_2 case $Q_i = -\frac{1}{2}\mathrm{i}c_{ijk}\nabla^j Z^{k\ell}\tilde{\Gamma}_\ell$

while in the $Spin_7$ case $Q_i = \frac{1}{4}c_{ijk\ell}\nabla^j Z^{k\ell mn}\Gamma_{mn}$

Generalized Structure groups and holonomy

- Although the ordinary Riemannian holonomy becomes generic for the corrected internal spaces, the supersymmetry preservation can still be understood on group theoretical grounds, using the notion of generalized holonomy.

- Consider the transverse groups generated by the generic Gamma matrix combinations present in the corrected Killing operator ($\Gamma_{[2]}$, $\Gamma_{[6]}$ and their closure), restricting attention to the D "transverse" dimensions only:
    - D=7 $SO(8)$
    - D=8 $SO(8)_+ \times SO(8)_-$
    - D=10 $SL(16, \mathbb{C})$

- Within these generalized transverse structure groups, the generalized holonomy is the group actually generated by the operators present in the corrected Killing spinor operator for a given space. Under decomposition into representations of these groups, the spinor representation contains a singlet, indicating continued supersymmetry preservation:
  - D=7 $SO(8) \to SO(7)$ (corrected G_2)
    $8_\pm \to 7 \oplus 1$
  - D=8 $SO(8)_+ \otimes SO(8)_- \to SO(8)_+ \otimes (\mathrm{Spin}_7)_-$ (corrected Spin_7)
    $(8,1) \oplus (1,8) \to (8,1) \oplus (1,7) \oplus (1,1)$
  - D=10 $SL(16,\mathbb{C}) \to [U(1) \times SL(5,\mathbb{C}) \times SL(5,\mathbb{C})] \ltimes [\mathbb{C}_1^{(10,1)} \oplus \mathbb{C}_3^{(10,5)}]$

    16 rep once again decomposes including a singlet