Complex submanifolds and holonomy

joint work with A.J. Di Scala and C. Olmos

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Contents

1 Main results

Main results

[–, Di Scala]

• computed the holonomy group $\Phi^{\perp}$ of the normal connection of complex symmetric submanifolds of $\mathbb{C}P^n$.

• as a by-product, given a new proof of the classification of complex symmetric submanifolds of $\mathbb{C}P^n$ by using a normal holonomy approach

Then, we prove Berger type theorems for $\Phi^{\perp}$, namely,

[–, Di Scala, Olmos]
M full, irreducible and complete

1. for $\mathbb{C}^n$, $\Phi^{\perp}$ acts transitively on the unit sphere of the normal space;
2. for $\mathbb{C}P^n$, if $\Phi^{\perp}$ does not act transitively, then M is the complex orbit, in the complex projective space, of the isotropy representation of an irreducible Hermitian symmetric space of rank greater or equal to 3.

2 Submanifolds and Holonomy

2.1 Real submanifold geometry

Submanifolds of real space forms

$M \hookrightarrow \mathbb{R}^n, S^n, \mathbb{R}H^n$ with induced metric $\langle\, ,\rangle$ and Levi-Civita connection ∇

νM: normal bundle of M with the normal connection $\nabla^{\perp}$

$\nu_0 M =$ *maximal parallel and flat subbundle of* νM

Notation
α second fundamental form
A shape operator
$R^{\perp}$ normal curvature tensor

recall $\langle \alpha(X,Y), \xi\rangle = \langle A_{\xi}X, Y\rangle$, which is symmetric in X, Y

Fundamental equations

Gauss: $\langle R_{X,Y}Z, W\rangle = \langle \alpha(X,W), \alpha(Y,Z)\rangle - \langle \alpha(X,Z), \alpha(Y,W)\rangle$
Codazzi: $(\bar{\nabla}_X \alpha)(Y,Z)$ are symmetric in X, Y, Z
Ricci: $\langle R^{\perp}_{X,Y}\xi, \eta\rangle = \langle [A_{\xi}, A_{\eta}]X, Y\rangle$

Nullity: $\mathcal{N} = \cap_{\xi} \ker A_{\xi}$

2.2 Normal holonomy for submanifolds of real space forms

Normal holonomy for submanifolds of real space forms

(Restricted) Normal Holonomy $\Phi^{\perp}$ ($\Phi^{\perp *}$):
(restricted) holonomy of the normal connection
on the normal bundle of a submanifold

Normal Holonomy Theorem [Olmos]
M submanifold of a space form $\overline{M}$.
$\Longrightarrow$ $\Phi^{\perp *}$ (at some point p) is compact,

$\Phi^{\perp *}$ acts (up to its fixed point set) as the isotropy representation of a Riemannian symmetric space (**s-representation**)

Consequences:
The Normal Holonomy Theorem is a very important tool for the study of submanifold geometry, especially in the context of *submanifolds with "simple extrinsic geometric invariants"*

e.g., isoparametric and homogeneous submanifolds

Distinguished class:
orbits of s-representations = flag manifolds

similar rôle as symmetric spaces in Riemannian geometry

Special cases

Symmetric submanifolds: characterizations **[Ferus, Strübing]**
– parallel second fundamental form $(\nabla\alpha = 0)$
– distinguished orbits of s-repr. (symmetric R-spaces)

K compact Lie group
$M = \mathrm{Ad}(K)X \cong K/K_X \hookrightarrow (\mathfrak{k}, -B(\,,\,))$

standard immersion of a cx flag manifold = *cx orbit of s-repr*

2.3 Complex submanifolds

Complex submanifolds
$M \hookrightarrow \mathbb{C}^n, \mathbb{C}P^n, \mathbb{C}H^n$ complex submanifold

J: complex structure (both on M and on the ambient space)

$$\alpha(X, JY) = J\alpha(X,Y) \iff A_\xi J = -JA_\xi = -A_{J\xi}$$

$$\implies [A_\xi, A_{J\eta}] = J[A_\xi, A_\eta] - 2JA_\xi A_\eta$$

for $\eta = \xi$, by the Ricci equation

$$\langle R^\perp(X,Y)\xi, J\xi\rangle = \langle -2JA_\xi^2 X, Y\rangle$$

Consequence: [Di Scala]
$M \hookrightarrow \mathbb{C}^n$ is full (not contained in any proper affine hyperplane) $\iff$ $\nu_0 M$ is trivial

[Indeed if ξ is a section of $\nu_0 M$, $R^\perp(X,Y)\xi = 0 \implies A_\xi = 0 \implies M$ not full]

2.4 Normal holonomy for submanifolds of complex space forms

Normal holonomy for complex (Kähler) submanifolds

- $M \hookrightarrow \mathbb{C}^n$

[Di Scala]: M is **irreducible** (up a totally geodesic factor) $\iff$
$\Phi^\perp$ **acts irreducibly**.

(extrinsic analogue of the de Rham decomposition theorem)

- $M \hookrightarrow \mathbb{C}P^n, \mathbb{C}^n, \mathbb{C}H^n$

Theorem [Alekseevsky-Di Scala]
If $\Phi^\perp$ acts irreducibly on $\nu_p M \implies$
$\Phi^\perp$ is linear isomorphic to the holonomy group of an irreducible Hermitian symmetric space.

M full & $\mathcal{N} = \{0\} \implies \Phi^\perp$ acts irreducibly

Homogeneous Kähler submanifolds

Calabi rigidity theorem of complex submanifolds $M \hookrightarrow \mathbb{C}P^N \implies$ isometric and holomorphic immersions are equivariant: any intrinsic isometry can be extended to $\mathbb{C}P^N$.

Borel-Weil construction

G simple Lie group, d positive integer

$\rho : G^{\mathbb{C}} \to \mathfrak{gl}(\mathbb{C}^{N_d+1})$ irreducible representation of $G^{\mathbb{C}}$ with highest weight $d\Lambda_j$

(Λ_j fundamental weight corresponding to the simple root α_j)

Induces a unitary representation of G

$$\implies \qquad M := G \cdot [p] \subset \mathbb{C}P^{N_d}$$

with p highest weight vector corresponding to $d\Lambda_j$

$\rightsquigarrow$ a full holomorphic embedding

$$f_d : M = G/K \hookrightarrow CP^{N_d}$$

d-th canonical embedding of M

Homogeneous Kähler submanifolds

M is the unique complex orbit of the action of G on $\mathbb{C}P^{N_d}$
(or equivalently, the unique compact orbit of the $G^{\mathbb{C}}$-action)

The induced metric on $M \subset \mathbb{C}P^{N_d}$ is Kähler-Einstein.

Calabi rigidity $\implies$ **any f_d factors through the Veronese embeddings and the first canonical embedding f_1,**

$$\text{i.e., } \boxed{f_d = \mathrm{Ver}_d \circ f_1}$$

where $\mathrm{Ver}_d : \mathbb{C}P^{N_1} \to \mathbb{C}P^{N_d}$ is the Veronese embedding

$$[z_0 : \cdots : z_{N_1}] \mapsto \left[z_0^d : \cdots : \sqrt{\frac{d!}{d_0! \dots d_{N_1}!}} z_0^{d_0} \dots z_{N_1}^{d_{N_1}} : \cdots : z_{N_1}^d \right]$$

($d_0, \dots, d_{N_1}$ range over all non-negative integers with $d_0 + \cdots + d_{N_1} = d$)

Symmetric complex submanifolds $M \subset \mathbb{C}P^n$

$$\boxed{M \subset \mathbb{C}P^n \text{ symmetric} \iff \nabla\alpha = 0}$$

Symmetric complex submanifolds $M \subset \mathbb{C}P^n$ were classified by Nakagawa-Takagi

Arise as *unique complex orbits in $\mathbb{C}P^n$ of the isotropy representation of an irreducible Hermitian symmetric space*

[–, Di Scala]

• computed the holonomy group of the normal connection of complex symmetric submanifolds of the complex projective space.

• as a by-product, given a new proof of the classification of complex symmetric submanifolds by using a normal holonomy approach

Symmetric complex submanifolds $M \subset P(T_{[K]}G/K)$

[–, Di Scala]:

Idea of the proof

Use [Alekseevsky-Di Scala] to get

Lemma 1. *$M = G/K$ Hermitian symmetric space*

$M \hookrightarrow \mathbb{C}P^N$ full embedding with $\nabla\alpha = 0$

$\implies \exists$ an irreducible Hermitian symmetric space H/S such that
$\Phi_p^\perp = S = K/I$ where $I \subset K$ is a normal subgroup,
$\dim_{\mathbb{C}}(\nu_p(M)) = \dim_{\mathbb{C}}(H/S)$ and
$\Phi_p^\perp$ acts on $\nu_p(M)$ as the isotropy repr. of S on $T_{[S]}(H/S)$.

$\rightsquigarrow$ computation of the 3rd column in the Table

Hermitian symmetric space G/K	M as (unique) complex K-orbit	Normal holonomy	Remarks
$\frac{E_7}{T^1 \cdot E_6}$	$\frac{E_6}{T^1 \cdot Spin_{10}}$	$\frac{SO(12)}{T^1 \cdot SO(10)}$	
$\frac{E_6}{T^1 \cdot Spin_{10}}$	$\frac{SO(10)}{U(5)}$	$\frac{U(6)}{U(5)}$	
$\frac{Sp(n+1)}{U(n+1)}$	$\mathbb{C}P^n$	$\frac{Sp(n)}{U(n)}$	Veronese
$Gr_2^+(\mathbb{R}^{n+2}) := \frac{SO(n+2)}{T^1 \cdot SO(n)}$	$Gr_2^+(\mathbb{R}^n)$	$\frac{U(2)}{U(1)}$	Quadrics
$\frac{SO(2n)}{U(n)}$	$Gr_2(\mathbb{C}^n)$	$\frac{SO(2(n-2))}{U(n-2)}$	Plücker
$Gr_a(\mathbb{C}^{a+b}) := \frac{SU(a+b)}{S(U(a)\times U(b))}$	$\mathbb{C}P^{a-1} \times \mathbb{C}P^{b-1}$	$\frac{SU(a+b-2)}{S(U(a-1)\times U(b-1))}$	Segre
	has $\nabla\alpha = 0$ and all symmetric subm arise in this way	Hermitian symmetric space whose isotropy representation gives the normal hol. action	

Alternate proof of classif. of cx symmetric subm of $\mathbb{C}P^N$

A tool is

Theorem 2. *Let $f_d : G/K \to \mathbb{C}P^{N_d}$ the d-th canonical embedding of G/K.*
If $\nabla\alpha = 0$ & f_d is not the Veronese embedding, f_d is the first canonical embedding f_1

$\Longrightarrow$ look at 1st canonical embeddings only.
The following theorem gives a sharp description.

Theorem 3. *If the first canonical embedding f_1 of an irreducible Hermitian symmetric space M of higher rank (≥ 1) has $\nabla\alpha = 0$.*
$\Longrightarrow$ *rank* $(M) = 2$.

Remark: list of images of the 1st can. embedding of an irred. Hermitian symm. space of rank two
$=$
list of the unique complex orbits of the isotropy action on the projective space $\mathbb{C}P(T_{[K]}G/K)$
(2nd column in the Table)

Higher canonical embedding and holonomy

Theorem [–, Di Scala]
Let $f_d : G/K \hookrightarrow \mathbb{C}P^{N_d}$ be the $d-$th canonical embedding of an irreducible Hermitian symmetric space. If $d \geq 2$ then the normal holonomy group is the full unitary group of the normal space (unless it is the Veronese embedding Ver $_2$)

Motivated by the above theorem we have the following

Question
$M \hookrightarrow \mathbb{C}P^N$ complete (connected) and full (i.e. not contained in a proper hyperplane) complex submanifold.
is it true in general that if the normal holonomy group is not the full unitary group, then M has parallel second fundamental form?

The answer is YES

3 Geometry of parallel focal manifolds and holonomy tubes

A Berger type Theorem

[–, Di Scala, Olmos]
$M \hookrightarrow \mathbb{C}^n, \mathbb{C}P^n$ full, irreducible and complete

1. for $\mathbb{C}^n$, $\Phi^\perp$ acts transitively on the unit sphere of the normal space;
   $\Longrightarrow \Phi_p^\perp = U(\nu_p M)$, since it acts as an s-representation

2. for $\mathbb{C}P^n$, if $\Phi^\perp$ does not act transitively, then M is the complex orbit, in the complex projective space, of the isotropy representation of an irreducible Hermitian symmetric space of rank greater or equal to 3.
   ($\Longrightarrow$ it is extrinsic symmetric)

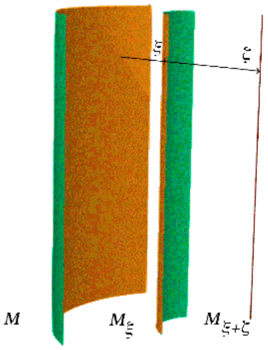
False if M is non complete (counterexamples)

The methods in the proofs rely heavily on the singular data of appropriate holonomy tubes (after lifting the submanifold to the complex Euclidean space, in the $\mathbb{C}P^n$ case) and basic facts of complex submanifolds.

Some geometry needed in the proof

Endpoint map

$$t_\xi : \quad M \to \mathbb{R}^n$$
$$x \mapsto x + \xi(x) = \exp(\xi(x))$$



$\boxed{\text{focal point}}$ in direction ξ = critical value of t_ξ.

$x + \xi(x)$ focal point in dir. of $\xi \iff \ker(\mathrm{id} - A_{\xi(x)})$ is non trivial

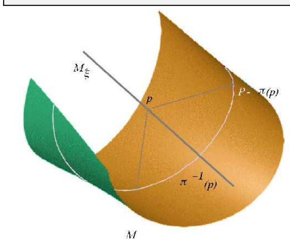
3.1 Parallel focal manifolds

Parallel focal manifolds

ξ parallel normal field, $\mathrm{im}(t_\xi) = \boxed{M_\xi = \{x + \xi(x) \mid x \in M\}}$

- if 1 is not an eigenvalue of A_ξ, parallel manifold
- if 1 is a constant eigenvalue of A_ξ, parallel focal manifold

$$T_xM = T_{x+\xi(x)}M_\xi \oplus \underbrace{\ker(\mathrm{id} - A_{\xi(x)})}_{\text{integrable}}$$



$\to$ one has a submersion
(non riemannian, in general)

$$\pi : M \to M_\xi : x \mapsto x + \xi(x),$$

$\pi^{-1}(p)$ isoparametric in $\nu_p M_\xi$
(by Olmos' Normal Holonomy Theorem)

3.2 Holonomy tubes

Holonomy tube

$$M_{\eta_p} = \{c(1) + \eta(1)\} = \{c(1) + \tau_c^\perp(\eta_p)\},$$

where $c : [0,1] \to M$ is an arbitrary curve starting at p and $\eta(t)$ is the $\nabla^\perp$-parallel transport of η_p along $c(t)$.

Proposition

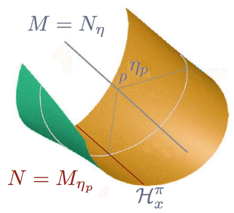
M_{η_p} has flat normal bundle (=: full holonomy tube)
$\iff \Phi^\perp_{\pi(p)}.(p - \pi(p))$ is maximal dimensional

$\mathscr{H}^\pi$ horizontal subspace of $\pi : N = M_{\eta_p} \to N_\eta = M$

tube formulae

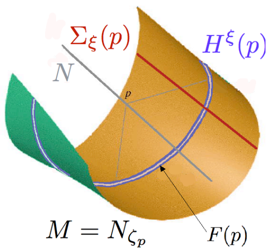
$$A^M_{\xi_x} = A^N_{\xi_x}(\mathrm{id} - A^N_{\eta(x)})^{-1}_{|\mathscr{H}^\pi_x}, \qquad \xi_x \in \nu_x N$$

$$A^N_{\xi_x|\mathscr{H}^\pi_x} = A^M_{\xi_x}(\mathrm{id} - A^M_{-\eta(x)})^{-1}_{|\mathscr{H}^\pi_x}, \qquad \xi_x \in \nu_x N$$



3.3 The canonical foliation

The canonical foliation



$N \hookrightarrow \mathbb{R}^n$, take $M = N_{\zeta_q}$ full holonomy tube

Assume:

- $\Phi^\perp$ acts **irreducibly** and **not transitively** on $\nu_p N$
- 0 is a constant eigenvalue of A^M_ξ,

  i.e. E^ξ_0 **is non-trivial**

canonical foliation

def. : $x \underset{\xi}{\sim} y$ if $\exists$ curve γ in M from x to y: $\dot{\gamma}(t) \perp E^\xi_0$, $\forall t$

$$\boxed{H^\xi(x) = \{y \in M : x \underset{\xi}{\sim} y\}}$$

the orthogonal distribution $\tilde{\nu}^\xi$ to the foliation $H^\xi(p)$ is integrable.
$\Sigma_\xi(x)$: leaf of $\tilde{\nu}^\xi$ through x

Note: $\tilde{\nu}^\xi \subseteq \mathscr{N} = \cap \ker A_\xi$ (nullity) $\Longrightarrow$ *the foliation is indep. on* $\xi \mid E^\xi_0 \neq \{0\}$

locally $\boxed{M = \bigcup_{x' \in \Sigma_\xi(x)} (H^\xi(x))_{x'-x}}$

The canonical foliation

$\mathscr{H}$: the horizontal distribution in M (w. r. to $\pi : M \to N$) $\Longrightarrow$

Technical Lemma

Assume that $\boxed{\exists \text{ parallel } \xi, \xi' \text{ such that } \mathscr{H} \subset (\ker A^M_\xi + \ker A^M_{\xi'})}$

$\Longrightarrow$ $\forall x \in M$, $H^\xi(x) = H^{\xi'}(x)$ is an isoparametric submanifold.

(we are around a generic point s. t. $(\ker A^M_\xi + \ker A^M_{\xi'})$ is a distribution of M)

Projecting down to N, $\boxed{N = \bigcup_{y \in \pi(\Sigma_\xi(x))} (\pi(H^\xi(x)))_{y-\pi(x)}}$

Using Thorbergsson Theorem

Corollary of the Technical Lemma

$\exists$ compact group K of isometries of $\mathbb{R}^n$ acting as an irred. s-representation s. t. (loc) $K \cdot \pi(x) = \pi(H^\xi(x))$, for all $x \in M$.

$\Longrightarrow$ N is locally given, around a generic point q, as

$$N = \bigcup_{v \in (\nu_0(K \cdot q))_q} (K \cdot q)_v.$$

Moreover the nullity space of N at p is $\mathscr{N}^N_p = (\nu_0(K \cdot p))_p$.

4 Complex submanifold geometry

4.1 Complex submanifolds of $\mathbb{C}^n$

Complex submanifolds of $\mathbb{C}^n$

$N \hookrightarrow \mathbb{C}^n$ **full, irreducible** complex submanifold for which $\Phi^\perp$ **does not act transitively** on the unit sphere of the normal space

Choose $\xi^1_q \in \nu_q N \mid \Phi^\perp_q \cdot [\xi^1_q] \in \mathbb{C}P(\nu_q N)$ (unique) complex orbit

$\Longrightarrow (\xi_q^1)^\perp \cap \nu_{\xi_q^1}\Phi_q^\perp \cdot \xi_q^1$ cx subsp. *(non trivial by non-transitivity!)*

Now choose $0 \neq \xi_q^2 \in (\xi_q^1)^\perp \cap \nu_{\xi_q^1}\Phi_q^\perp . \xi_q^1$

Since $R_{X,Y}^\perp \in L(\Phi_q^\perp)$, $0 = \langle R_{X,Y}^\perp \xi_q^1, \xi_q^2 \rangle \underset{\text{Ricci}}{=} \langle [A_{\xi_q^1}^N, A_{\xi_q^2}^N] X, Y \rangle$.
The same is true if we replace ξ_q^2 by $J\xi_q^2 \Longrightarrow [A_{\xi_q^1}^N, A_{J\xi_q^2}^N] = 0$.

$\boxed{\text{By complex geometry} \qquad A_{\xi_q^1}^N A_{\xi_q^2}^N = A_{\xi_q^2}^N A_{\xi_q^1}^N = 0}$

Take the holonomy tube $\boxed{M := (N_{\xi_q^1})_{\xi_q^2} = N_{\xi_q^1 + \xi_q^2}}$

$\xi_q^1, \xi_q^2 \rightsquigarrow$ parallel v. f. ξ, ξ' on M

Tube formula $\Longrightarrow A_\xi^M A_{\xi'}^M{}_{|\mathscr{H}} = 0 \Longrightarrow \mathscr{H} \subset (\ker A_\xi^M + \ker A_{\xi'}^M)$
$\Longrightarrow$ Technical Lemma and its corollary apply

Complex submanifolds of $\mathbb{C}^n$
$\Longrightarrow \exists$ compact group K of isometries of $\mathbb{C}^n$, which acts as the isotropy representation of an irreducible *Hermitian* symmetric space such that

$$\boxed{N \underset{\text{locally}}{=} \bigcup_{v \in (\nu_0(K \cdot q))_q} (K \cdot q)_v}$$

Moreover $\mathscr{N}_p^N = (\nu_0(K.p))_p$.

$\Longrightarrow$

Proof of the Berger-type Theorem for submanifolds of $\mathbb{C}^n$
We assume that 0 is the fixed point of K.

N is complete $\Longrightarrow$ if $p \in N$, the line $\{t \mapsto tp\} \subset N$

$\forall t$, $T_{tp}N = T_pN$, as subspaces of $\mathbb{C}^n \Longrightarrow$
the isotropy K_{tp} must leave this subspace invariant.

A contradiction for $t = 0$, since K acts irreducibly.
Thus the normal holonomy group must be transitive.

4.2 Complex submanifolds of $\mathbb{C}P^n$

Complex submanifolds of $\mathbb{C}P^n$
Let $M \hookrightarrow \mathbb{C}P^n$ be a full complex submanifold.

Consider $\pi : \mathbb{C}^{n+1} \setminus \{0\} \to \mathbb{C}P^n$

$\widetilde{M}$: lift M to $\mathbb{C}^{n+1} \setminus \{0\}$, i.e. $\boxed{\widetilde{M} := \pi^{-1}(M)}$

$\mathscr{V}$: vertical distribution of the submersion $\pi : \widetilde{M} \to M$.

It is standard to show that $\mathscr{V} \subset \mathscr{N}^{\widetilde{M}}$.

If X is a tang. vector to M we let $\widetilde{X}$ be its horiz. lift to $\mathbb{C}^{n+1} \setminus \{0\}$.
$\pi : \mathbb{C}^{n+1} \setminus \{0\} \to \mathbb{C}P^n$ is not a Riemannian submersion. Anyway,

O'Neill's type formula
Let $\widetilde{X}, \widetilde{Y} \in \Gamma(\mathbb{C}^{n+1} \setminus \{0\})$ be the horizontal lift of the vector fields $X, Y \in \Gamma(\mathbb{C}P^n)$. Then,

$$(D_{\widetilde{X}} \widetilde{Y})_{\widetilde{p}} = (\widetilde{\nabla_X^{FS} Y})_{\widetilde{p}} + \mathscr{O}(\widetilde{X}, \widetilde{Y})$$

where $\mathscr{O}(\widetilde{X}, \widetilde{Y}) \in \mathscr{V}_{\widetilde{p}}$ is vertical.

Complex submanifolds of $\mathbb{C}P^n$

Lemma 1
$M \subset \mathbb{C}P^n$, $\widetilde{M} \subset \mathbb{C}^{n+1}$ be its lift to $\mathbb{C}^{n+1}$.

Assume that the tangent vector $\widetilde{v}_{\widetilde{p}} \in T_{\widetilde{p}}\widetilde{M}$ is not a complex multiple of the position vector $\widetilde{p}$.
If $\widetilde{v}_{\widetilde{p}} \in \mathscr{N}^{\widetilde{M}} \Longrightarrow v_p \in \mathscr{N}^M$.

Lemma 2
Assume that $M \subset \mathbb{C}P^n$ is full and
$\Phi^{\perp M}$ does not act transitively on $\nu_p(M)$.

$\Longrightarrow \Phi^{\perp \widetilde{M}}$ does not act transitively on $\nu_{\widetilde{p}}(\widetilde{M})$, where $\pi(\widetilde{p}) = p$.

Important fact (special case of a Theorem in [Abe-Magid])
Let $M \subset \mathbb{C}P^n$ complete full with $\Phi^{\perp}$ not transitive
$\Longrightarrow \mathcal{N}^M = \{0\}$

Proof of the Berger-type Theorem for submanifolds of $\mathbb{C}P^n$

$N = \widetilde{M} \subset \mathbb{C}^{n+1} \Longrightarrow \widetilde{M} = \bigcup_{v \in (\nu_0(K \cdot q))_q} (K \cdot q)_v$
(K is the isotropy group of a irreducible Hermitian symmetric space)

Observe also that $\nu_0(K \cdot q)_q$ is a complex subspace ($=\mathcal{N}^N$))

Then Lemma 1 and special case of Abe-Magid $\Longrightarrow \dim_{\mathbb{C}}(\nu_0(K \cdot q)_q) = 1$, otherwise the nullity of the second fundamental form of M would be not trivial.

Since $\widetilde{M}$ is full $\Longrightarrow$ the unique fixed point of K is $0 \in \mathbb{C}^{n+1}$.

So the leaves of the nullity distribution $\mathcal{N}^{\widetilde{M}}$ are just the complex lines given by the fibers of the submersion $\pi : \widetilde{M} \to M$.

Thus, K acts transitively on the complex submanifold $M \subset \mathbb{C}P^n$.

Therefore, M is a complex orbit of the projectivization of an irreducible Hermitian s-representation.

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