$$L[y] = y^{(n)} + a_{n-1}(t)\, y^{(n-1)} + \cdots + a_0(t)\, y(t) = h(t)$$

Betrachte homogene Gleichung

$$L[y] = 0$$

$\Leftrightarrow$

$$y' = A(t)\, y \qquad y = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

$$y_1(t) = y(t)$$
$$y_2(t) = y'(t)$$
$$\vdots$$
$$y_n(t) = y^{(n-1)}(t)$$

$$\frac{d}{dt}\begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} y' \\ y'' \\ \vdots \\ y^{(n)} \end{pmatrix} = \begin{pmatrix} y_2(t) \\ y_3(t) \\ \vdots \\ y_n(t) \\ -a_{n-1} y_n - \cdots - a_0 y_1(t) \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & & 0 \\ & & & & \\ \cdot & - & - & - & 0 \; 1 \\ -a_0 & \cdot & - & & -a_{n-1} \end{pmatrix} \begin{pmatrix} y_1 \\ \\ \\ y_n \end{pmatrix}$$

$$L[y] = h \qquad \text{inhomoge}$$

$$\frac{d}{dt}\begin{pmatrix} y_1 \\ \vdots \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} 0 & 1 & & \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ & & 0 & 1 \\ -a_0 & & \cdots & -a_{n-1} \end{pmatrix}\begin{pmatrix} y_1 \\ \vdots \\ \vdots \\ y_n \end{pmatrix} + \begin{pmatrix} 0 \\ \vdots \\ \vdots \\ 0 \\ \boxed{h(t)} \end{pmatrix}$$

$$Y_p(t) = \int_{t_0}^{t} G(t,\tau)\, h(\tau)\, d\tau$$

$$G(t,\tau) = w(t-\tau+t_0)$$

w löst $L[w] = 0$

Anfangswerte $w^{(k)}(t_0) = \begin{cases} 0 & k \neq n-1 \\ 1 & k = n-1 \end{cases}$

$$Y_p'(t) = G(t,t)\, h(t) + \int_{t_0}^{t} \frac{\partial}{\partial t} G(t,\tau)\, h(\tau)\, d\tau$$

$$= w(t-t+t_0)\, h(t) + \int_{t_0}^{t} w'(t-\tau+t_0)\, \overset{h(\tau)}{\vee}\, d\tau$$

$$= \underbrace{w(t_0)}_{\substack{=0 \\ \text{falls } n>1}}\, h(t) + \int_{t_0}^{t} w'(t-\tau+t_0)\, h(\tau)\, d\tau$$

$$Y_p''(t) = w'(t_0)\, h(t) + \int_{t_0}^{t} w''(t-\tau+t_0)\, h(\tau)\, d\tau$$

$\vdots$

$$Y_p^{(n-1)}(t) = \underbrace{w^{(n-2)}(t_0)}_{=0}\, h(t) + \int_{t_0}^{t} w^{(n-1)}(t-\tau+t_0)\, h(\tau)\, d\tau$$

$$Y_p^{(n)}(t) = \underbrace{w^{n-1}(t_0)}_{1}\, h(t) + \int_{t_0}^{t} w^{(n)}(t-\tau+t_0)\, h(\tau)\, d\tau$$

$$L[y_p] = y^{(n)} + a_{n-1} y^{(n-1)} + \dots + a_0 y$$

$$= h(t) + \int_{t_0}^{t} w^{(n)}(t-\tau+t_0)\, h(\tau)\, d\tau$$

$$+ \int_{t_0}^{t} a_{n-1} w^{(n-1)}(t-\tau+t_0)\, h(\tau)\, d\tau$$

$$+$$

$$\vdots$$

$$+ \int_{t_0}^{t} w'(t-\tau+t_0)\, h(\tau)\, d\tau$$

$$+ \int_{t_0}^{t} w(t-\tau+t_0)\, h(\tau)\, d\tau$$

$$= h(t) + \int_{t_0}^{t} L[w](t-\tau+t_0)\, h(\tau)\, d\tau$$

$$= h(t) + \int_{t_0}^{t} 0 \; h(\tau)\, d\tau = h(t)$$

$p(x) = (x-2)^2 = x^2 - 4x + 4$

$L[y] = y^{(2)} - 4y^{(1)} + 4y^{(0)}$

$p(\lambda) = \lambda^2 - 4\lambda + 4$

$y(t) = e^{2t}$

$y'' - 4y' + 4y = 4e^{2t} - 8e^{2t} + 4e^{2t} = 0$

$y(t) = t e^{2t}; \quad y'(t) = e^{2t} + 2te^{2t};$

$y'' - 4y' + 4y = \cancel{2e^{2t}} + \cancel{2e^{2t}} + 4t e^{2t} - \cancel{4e^{2t}} - 8te^{2t}$
$+ 4te^{2t}$

$= 0$

$\lambda_k \quad r_k$ -fache Nullstelle

$e^{\lambda_k t}$

$t e^{\lambda_k t}$

$\vdots$

$t^{k-1} e^{\lambda_k t}$

λ Nst. eines reellen Polynoms p

$\lambda \notin \mathbb{R} \Rightarrow \bar{\lambda}$ Nst von p

$$p(\lambda) = (z-\lambda)^2 (z-\bar{\lambda})^2$$
$$= ((z-\lambda)(z-\bar{\lambda}))^2$$
$$= (z^2 - 2\,\mathrm{Re}\,\lambda\, z + \lambda\bar{\lambda})^2$$

$\lambda = 1+i$ $\qquad (1+i)(1-i) = 1^2 - (-i)^2 = 2$

$$p(\lambda) = (z^2 - 2z + 2)^2$$

$$= z^4 - 4z^3 + 8z^2 - 8z + 4$$

$$L[y] = y^{(4)} - 4y^{(3)} + 8y^{(2)} - 8y^{(1)} + 4y^{(0)}$$

$L[y] = 0$

$\mathrm{Re}\,\lambda = 1, \ \mathrm{Im}\,\lambda = 1$

$y_{1a}^c = \underline{e^t \cos t}$ $\qquad y_{1a}^s = e^t \sin t$

$y_2^c = t e^t \cos t$ $\qquad y_2^s = \underline{t e^t \sin t}$

$$Y_1^c = e^t \cos t$$

$$(Y_1^c)' = e^t \cos t - e^t \sin t$$

$$(Y_1^c)'' = e^t \cos t - e^t \sin t - e^t \sin t - e^t \cos t$$

$$= -2 e^t \sin t$$

$$(Y_1^c)''' = -2 e^t \sin t - 2 e^t \cos t$$

$$(Y_2^c)'''' = -2 e^t \sin t - 2 e^t \cos t - 2 e^t \cancel{\sin} t + 2 e^t \sin t$$

$$= -4 e^t \cos t$$

$$\cancel{-4 e^t \cos t} + \cancel{8 e^t \sin t} + \cancel{8 e^t \cos t} - \cancel{16 e^t \sin t} - \cancel{8 e^t \cos t}$$

$$+ \cancel{8 e^t \sin t} + \cancel{4 e^t \cos t}$$

$$= 0$$