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Differential Topology

Problem Set 6

1. Let $f : M \to N$ be a smooth map which is transverse to a submanifold $Z \subseteq N$ and its boundary $\partial Z \subset N$. Prove that $f^{-1}(Z) \subseteq M$ is a submanifold of dimension $\dim M + \dim Z - \dim N$ with boundary $f^{-1}(\partial Z)$.

2. Let M be a smooth manifold, possibly with boundary.

a) If $n = \dim M > 1$, then M is orientable if and only if there exists an atlas for M all of whose transition maps are orientation preserving diffeomorphisms between open subsets of $\mathbb{R}^n$.

b) What happens if $\dim M = 1$?

3. Let M be a compact manifold with boundary $\partial M = B$, and let N be a connected manifold with $\dim N = \dim B$. Prove that any smooth map $f : B \to N$ which extends to a smooth map $F : M \to N$ has mod 2 degree 0. Moreover, if M and N are oriented (and B inherits the boundary orientation from M), then the integer valued degee of f is also zero.

4. **a)** Let $f : M \to N$ and $g : N \to P$ be smooth maps between closed connected oriented manifolds of equal dimension. Prove that

$$\deg(f \circ g) = \deg(f) \cdot \deg(g).$$

b) Prove that any smooth map between closed connected oriented manifolds which is a homotopy equivalence has degree ± 1.

5. What is the degree of

a) the map $f_k : S^1 \to S^1$ given by $f(z) = z^k$ with $k \in \mathbb{Z}$? (Here we view $S^1 \subseteq \mathbb{C}$ as the unit circle in $\mathbb{C}$.)

b) the map $\mathbb{R}^2 \cup \{\infty\} \to \mathbb{R}^2 \cup \{\infty\}$ defined by a rational function $f(z) = \frac{p(z)}{q(z)}$, where p and q are polynomials of degrees n and m without common roots?