Universität Hamburg
Janko Latschev

Winter 2017/18

Differential Topology

Problem Set 3

1. Prove that polynomials are dense in $C^r(\mathbb{R}, \mathbb{R})$ with the weak topology for every $r \geq 0$ but not dense for any $r \geq 0$ in the strong topology.

2. Let $r \geq 0$ and let $A \subsetneq M$ be a closed subset of a C^r manifold M. Prove:

a) There exists a C^r function $f : M \to [0, \infty)$ with $f^{-1}(0) = A$.

b) If $B \subseteq M \setminus A$ is another closed subset, then there exists a C^r function $g : M \to [0, 1]$ such that $g^{-1}(0) = A$ and $g^{-1}(1) = B$.

3. Let $U \subseteq \mathbb{R}^n$ be open and $\{\lambda_0, \lambda_1\}$ a partition of unity subordinate to the covering $U = U_0 \cup U_1$ by open sets. Prove that for every $r \geq 0$ and every fixed map $f \in C^r(U, \mathbb{R}^m)$ the mapping

$$\begin{aligned} G : C^r(U, \mathbb{R}^m) &\to C^r(U, \mathbb{R}^m) \\ g &\mapsto \lambda_0 \cdot g + \lambda_1 \cdot f \end{aligned}$$

is continuous in the strong C^r topology.