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Differential Topology

Problem Set 1

1. Let $U \subset \mathbb{R}^n$ be a connected open subset, and let $p : U \to U$ be a C^r map such that $p \circ p = p$. Prove that the subset $F \subset U$ of fixed points of p forms a C^r submanifold of $\mathbb{R}^n$.

2. Let M be a differentiable manifold (of some class C^r, $r \geq 1$), and let $\tau : M \to M$ be a fixed point free involution of the same class, i.e. $\tau(p) \neq p$ for all $p \in M$ and $\tau \circ \tau = \mathrm{id}_M$.

a) Prove that the quotient space M/τ which is obtained by identifying every point with its image under τ is a topological manifold, and it admits a unique C^r-structure for which the projection map $\pi : M \to M/\tau$ is a local diffeomorphism.

b) Give examples of this phenomenon.

3. Prove that the subset

$$Q := \{\mathbf{z} = (z_1, \dots, z_n) \in \mathbb{C}^n \mid \sum_{j=1}^{n} z_j^2 = 1\} \subset \mathbb{C}^n \cong \mathbb{R}^{2n}$$

is diffeomorphic to the tangent bundle of S^{n-1}.

4. Prove or disprove:

a) There is an immersion of the punctured torus $S^1 \times S^1 \setminus \{pt\}$ into $\mathbb{R}^2$.

b) Any product of spheres admits an embedding into euclidean space with codimension 1.