Universität Hamburg
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Fachbereich Mathematik
Algebra und Zahlentheorie
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# Syllabus for the course on Lie Algebras, Sommerterm 2018

## Basic theory

Lie Algebra, Jacobi identity, examples, semi-direct product, ideals, extensions of Lie algebras, derivations, $\mathrm{ad}_x$,

## Abstract algebra

free magma, tensor algebra, free algebra, free Lie algebra (isomorphic descriptions), Hall families, filtered vectorspaces, graded vectorspaces, universal envelopping algebra, diagonal map, primitive elements, symmetric algebra, Poincare-Birkhoff-Witt theorem,

## $\mathfrak{g}$-modules

$\mathfrak{g}$-action, relations to $\mathcal{U}(\mathfrak{g})$-modules, invariant bilinear form, Killing form, trace, tensor products

## Nilpotent/Solvable Lie Algebras

derived series, Engel theorem, central series, Lie theorem, Cartan Criterion, decomposition

## Semi-simple Lie algebras

non-degenerate Killing form, radical, simple Lie algebras, complete reducibility, Weyl's theorem, derivations, Levi's theorem, decomposition, tensor invariants

## $\mathfrak{sl}_2$ and its representations

$\mathfrak{sl}_2 \cong \mathfrak{n}_- \oplus \mathfrak{h} \oplus \mathfrak{n}_+$, roots, weights, eigenvector, multiplicity, Borel subalgebra, primitive element, irreducible modules, examples, heighest weight,

## Complex semi-simple Lie algebras

Cartan subalgebra, root system, decomposition, root spaces $\mathfrak{g}_\alpha$, $\mathfrak{h}_\alpha$, Weyl's & Serre's relation, simple root system, Coxeter graph, Cartan matrix, Dynkin Diagram

## References

J.-P. Serre Lie Algebras
J.-P. Serre: Complex semisimple Lie Algebras
J. Humphreys: Introduction to Lie Algebras and Representation Theory
Erdmann: Introduction to Lie Algebras
Bourbaki: Lie algebras and Lie groups.



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