Modeling and Performance Analysis of a Node in Fault Tolerant Wireless Sensor Networks

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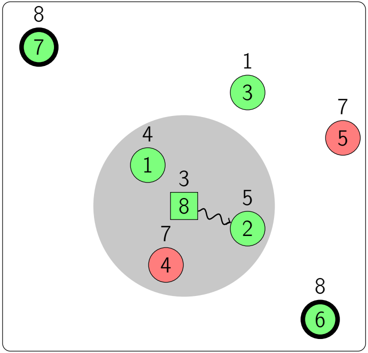
Technical problem

Problem settings

- wireless network of sensor nodes
- network changes randomly in time
- short range
- failures and delays

Technical problem

Transmit measured data from nodes to sink nodes.



More specific problem

Understand the long term behavior of the network.

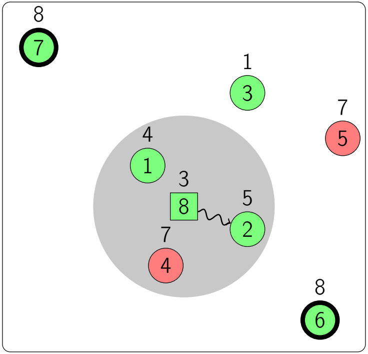
Challenges

Complexity of network, changing of network topology and environment.

Network

Important features

- data sources: (sensors, other nodes, redundant messages)
- one-hop neighborhood
- delivery value (delivery probability)
- failure rate
- time out
- active / sleeping



referenced node

sink node

data transmission

other node

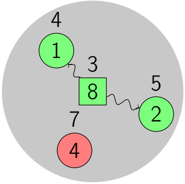
active/sleeping

one-hop neighborhood

Sending message strategies

Direct transmission to sink nodes (wait for sink)

- pro: low data traffic, low power consumption
- contra: long delays, not fault tolerant



Simple flooding (send data to everyone)

- pro: fault tolerant, shorter delays
- contra: high traffic overhead, high power consumption

Better solution:

Transmit "sufficient" number of message copies:

Using **nodal delivery value** (delivery probability) decide

- **where to send** message (to node with high delivery value)
- (**whether to keep** another **copy** of message)

Nodal delivery values

The **nodal delivery values** are on a scale

- $D := \{1, 2, \ldots, d\}$, $d < \infty$
  The higher nodal delivery value, the higher nodal delivery probability.
- Nodal delivery values change in time.

Updating:

- $\eta \equiv$ maximal delivery value in neighborhood and
- $\xi \equiv$ delivery value of the referenced node (RN),

whenever RN has send a packet to a node with delivery value η,
it updates its own delivery value ξ as follows

$$\xi \to \begin{cases} \xi + 1, & \text{if } \xi < \eta; \\ \xi - 1, & \text{if } \xi > \eta; \\ \xi, & \text{if } \xi = \eta. \end{cases}$$

Construction of the mathematical model

- Construct Markov process $Z = (Z(t) : t \in [0, \infty))$ on suitable state space E.
- Sojourn times are exponential.
- Find limiting and steady state distribution $\pi(z) := \lim_{t \to \infty} P(Z(t) = z)$.
- Use π to compute long time performance of the system.

Queue $X(t)$

Queue represents a list of messages arrived at the node with rate λ and processed with rate $\mu(1 - f)$ in FCFS order.
f is the failure rate.

Environment $Y(t)$

Additional parameters of the system

From network to single node

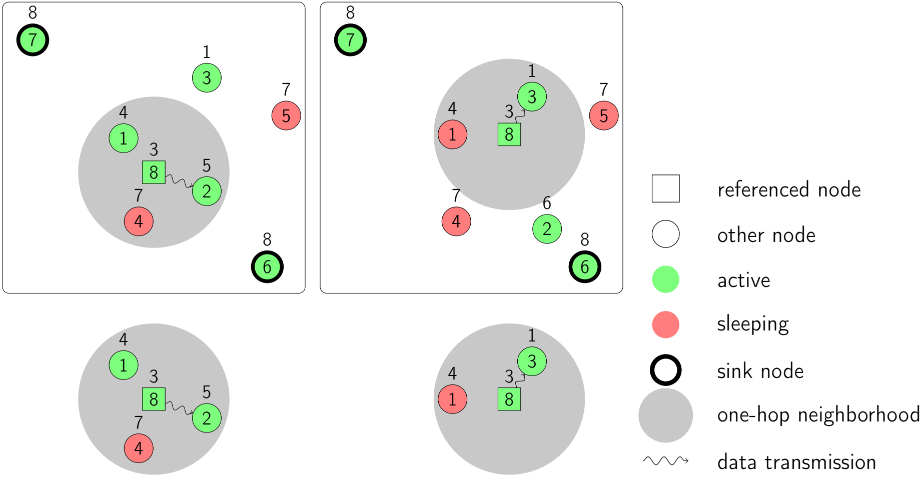


Figure: The numbers within circles or rectangle are node indices, the numbers above are current delivery values.

Pure queueing model

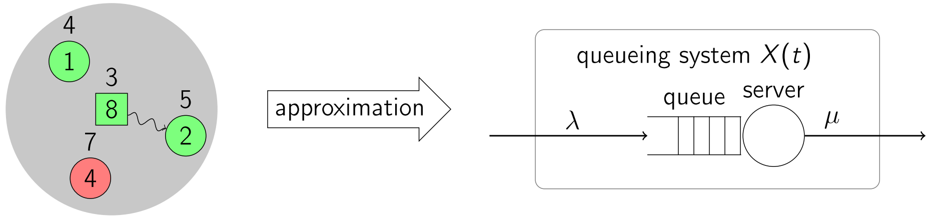


Figure: From node to queue, $\lambda \equiv$ input rate, $\mu \equiv$ service rate

Standard tools: **M/M/1/∞-queue** with

$$\lim_{t \to \infty} P(X(t) = n) = \left(1 - \frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^n$$

- advantage: simple
- disadvantage: too simple

Queue with environment

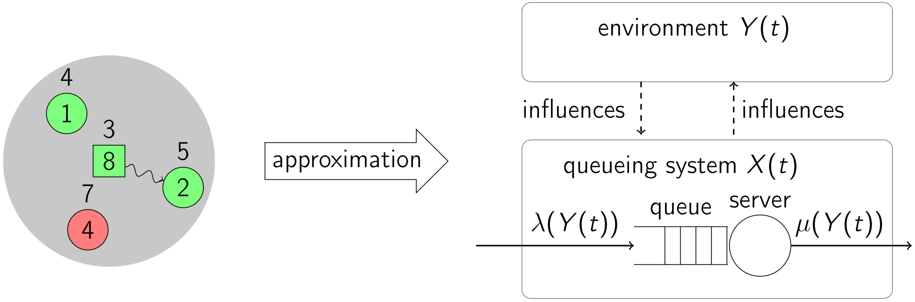


Figure: From node to queue with environment.

Standard tools: **matrix geometrical methods**

- advantage: modeling of complex behavior
- disadvantage: in general no closed analytical solution

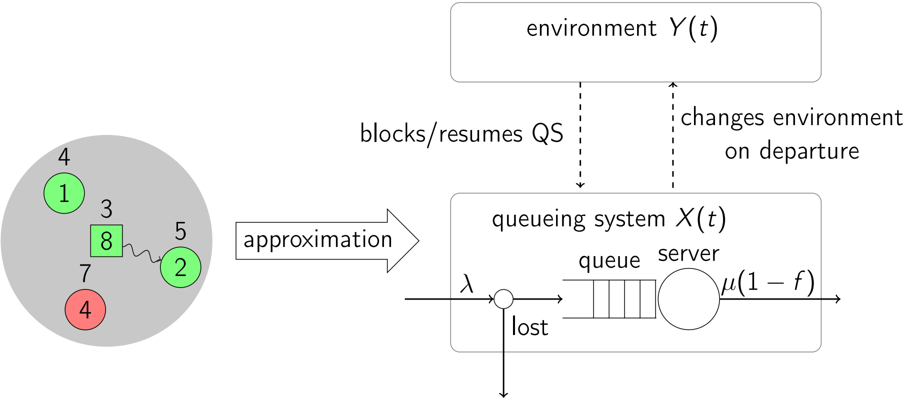
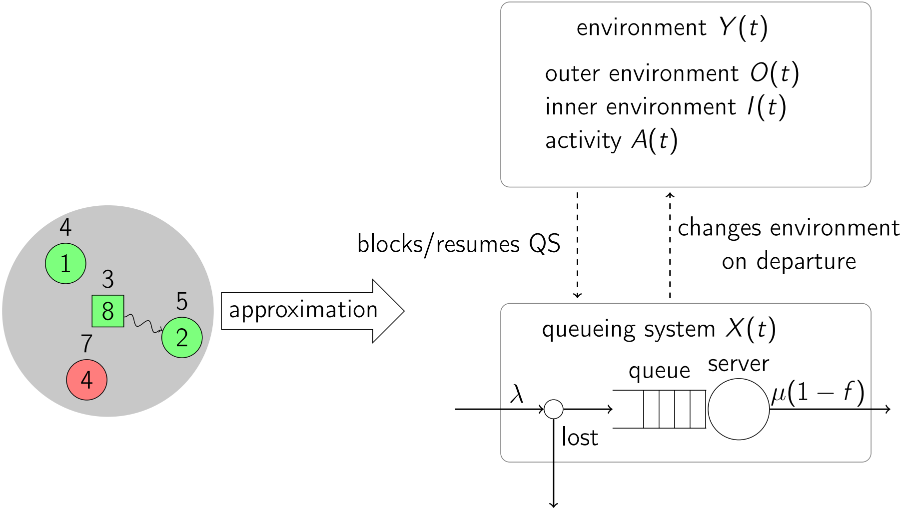


Figure: Our model: queue with environment.


4
1
3
8
5
2
7
4
approximation
blocks/resumes QS
changes environment
on departure
environment $Y(t)$
outer environment $O(t)$
inner environment $I(t)$
activity $A(t)$
queueing system $X(t)$
queue
server
λ
$\mu(1-f)$
lost

Describes the development of N nodes in one-hop neighborhood of RN.
Irreducible Markov process

$$O = (O(t) : t \geq 0),$$

with state space

$$E_o := (\{0\} \cup D)^N$$

$0 \equiv$ "node is not available for RN".

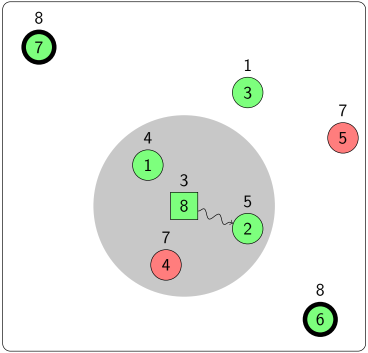


Figure: $O(t) = (4, 5, 0, 0, 0, 0, 0)$

The inner (local) environment of RN is a stochastic process, which is not Markov for its own

- $\Delta := \{1, \dots, T\}$, timer
- $D := \{1, 2, \dots, d\}$, delivery value

$$I = (I(t) : t \geq 0),$$

with state space $E_i := \Delta \times D$.

On timeout:

$$\xi \to \begin{cases} \xi - 1, & \text{if } \xi > 1; \\ \xi, & \text{if } \xi = 1. \end{cases}$$

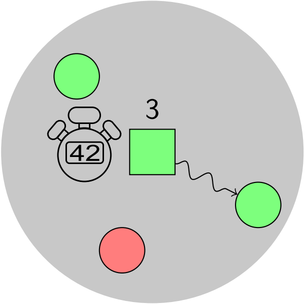


Figure: $I(t) = (42, 3)$

Activity

- $E_a := \{0, 1\}$
- $0 \equiv$ "sleeping" , $1 \equiv$ "active"

$$A = (A(t) : t \geq 0), \qquad \text{with state space} \quad E_a$$

The dwell time in the active status is exponential-α, whereas in sleep status exponential-β.

Queue

- $\mathbb{N}_0 \equiv$ number of messages in the referenced node
- $\lambda \equiv$ input rate (sources: sensors and other nodes)
- $\mu \equiv$ service rate
- $f \equiv$ constant failure rate (source for redundant messages)

Queue length process:

$$X = (X(t) : t \geq 0), \qquad \text{with state space } \mathbb{N}_0,$$

Queue - environment interactions

- no input and no service when sleeping $\equiv A(t) = 0$
- no input and no service when outer environment is quiet $\equiv O(t) = (0, 0, ..., 0)$
- after message successfully transmitted nodal delivery value is adopted and timer is reseted $(I(t))$

Steady state results

The process $Z := (X, Y) = (X, A, I, O)$ is a homogeneous Markov process with queueing process X and environmental process $Y := (A, I, O)$

Theorem

Z is ergodic on E iff $\lambda < \mu(1-f)$. Then its unique stationary distribution π fulfills for all $(n, a, t, \xi, \eta) \in \mathbb{N}_0 \times \underbrace{\{0,1\} \times \Delta \times D \times (\{0\} \cup D)^N}_{E_Y}$

$$\pi(n, a, t, \xi, \eta) = \left(1 - \frac{\lambda}{\mu(1-f)}\right)\left(\frac{\lambda}{\mu(1-f)}\right)^n \cdot \theta(a, t, \xi, \eta),$$

where θ is a (unique) probability distribution on E_Y. Solution of $\theta Q_{red} = 0$, $Q_{red} \in \mathbb{R}^{E_Y \times E_Y}$. θ independent from μ and f.

- $n \equiv$ queue length
- $a \equiv$ activity
- $t \equiv$ timer state
- $\xi \equiv$ local delivery value
- $\eta \equiv$ outer delivery value and availability

System performance

$f(n, a, t, \xi, \eta)$ abstract performance metric per time per state

$$\frac{1}{T}\int_0^T f\left(X_t, A_t, I_t, O_t\right) dt \overset{T\to\infty}{\longrightarrow} \sum_{(n,k)} f(n, a, t, \xi, \eta)\pi(n, a, t, \xi, \eta),\ a.s.$$

- $n \equiv$ queue length
- $a \equiv$ activity
- $t \equiv$ timer state
- $\xi \equiv$ local delivery value
- $\eta \equiv$ outer delivery value and availability

Reduction of environment

From (X, A, I, O) to (X, A, I). Approximate I by a random walk

$$\xi \to \begin{cases} \xi + 1, & \text{with probability } r^{+}(\xi) \quad \text{if } \xi < d; \\ \xi - 1, & \text{with probability } r^{-}(\xi) \quad \text{if } \xi > 1; \\ \xi, & \text{with probability } \ldots \end{cases}$$

Corollary (part I)

Iff Z is ergodic, then its unique stationary distribution π fulfills

$$\pi(n, a, t, \xi) = \left(1 - \frac{\lambda}{\mu(1-f)}\right)\left(\frac{\lambda}{\mu(1-f)}\right)^{n} \cdot \theta(a, t, \xi)$$

$$\theta(a, t, \xi) = \left(\frac{\beta}{\alpha+\beta}\right)^{a} \cdot \left(\frac{\alpha}{\alpha+\beta}\right)^{1-a} \cdot \left(\frac{\delta}{\lambda+\delta}\right)^{T-t} K_{\Delta}^{-1} \cdot \psi(\xi)\,.$$

$K_{\Delta} := \frac{\delta}{\lambda}\left(\frac{\lambda+\delta}{\delta} - \left(\frac{\delta}{\lambda+\delta}\right)^{T-1}\right)$, *$\psi$ is a probability on D.*

The marginal nodal delivery value distribution ψ is calculated recursively.

Nodal delivery value distribution $\psi(\xi)$

$$\pi(n,a,t,\xi) = \left(1 - \frac{\lambda}{\mu(1-f)}\right)\left(\frac{\lambda}{\mu(1-f)}\right)^n \cdot \theta(a,t,\xi)$$

$$\theta(a,t,\xi) = \left(\frac{\beta}{\alpha+\beta}\right)^a \cdot \left(\frac{\alpha}{\alpha+\beta}\right)^{1-a} \cdot \left(\frac{\delta}{\lambda+\delta}\right)^{T-t} K_\Delta^{-1} \cdot \psi(\xi)\,.$$

Corollary (part II)

$$\begin{aligned}
&\psi(\xi) \\
= \;& \psi(\xi)\left(\frac{\delta}{\lambda+\delta}\right)^T 1_{(\xi=1)} + \psi(\xi-1)\left(1-\left(\frac{\delta}{\lambda+\delta}\right)^T\right) r^+(\xi-1) 1_{(\xi>1)} \\
&+\psi(\xi+1)\left(\frac{\delta}{\lambda+\delta}\right)^T 1_{(\xi<d)} + \psi(\xi+1)\left(1-\left(\frac{\delta}{\lambda+\delta}\right)^T\right) r^-(\xi+1) 1_{(\xi<d)} \\
&+\psi(\xi)\left(1-\left(\frac{\delta}{\lambda+\delta}\right)^T\right)[1 - r^+(\xi)1_{(\xi<d)} - r^-(\xi)1_{(\xi>1)}]
\end{aligned}$$

Further extension of environment

More general environment and interactions are possible

Interaction rules

- environment $\to$ queue
  - blocking input and service simultaneously
- queue $\to$ environment
  - changing of the environment after the service

Theorem

As long as the ***interaction rules*** *hold the limiting distribution*

$$\pi(n, k) = \pi_X(n)\theta(k)$$

- $n \equiv$ *message queue length*
- $k \equiv$ *environment state (additional parameters)*

Critics and open questions

"Low cost" improvement of $M/M/1/\infty$ model.

Critics

- Up to now we have no comparison with real word results.
- No data is gathered when the surrounding nodes are unavailable.
- Failure rate is not adapted to changes in the environment.

Open questions and ongoing research

- Compare the model with real systems.
- Can the model be used to bound more complex models?
- Can the results be used as Initial value for simulating more complex networks, even with non-product form steady state model?
- Extend results to networks.

Thank you for your attention!

Bibliography

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