**Homework 9**, due Tuesday 6 May, 12:00

1. Exercise 46 on p. 31 of the syllabus. [4 pts]

2. Complete the proof of Theorem 50, i.e., show that the definition of $\rightarrow$ from Definition 48 satisfies conditions 1–4:

   *1.* $a \rightarrow a = \top$

   *2.* $a \wedge (a \rightarrow b) = a \wedge b$

   *3.* $b \wedge (a \rightarrow b) = b$

   *4.* $a \rightarrow (b \wedge c) = (a \rightarrow b) \wedge (a \rightarrow c)$ [4pts]

3. (a) Exercise 55 (4): Show that if a frame $\mathfrak{F}$ is rooted, then the corresponding Heyting algebra has a second-greatest element. [2 pts]

   (b) Exercise 59 (4): Show that if a Heyting algebra $\mathfrak{A}$ has a second-greates element, then the corresponding Kripke frame is rooted. [2 pts]

4. Show that in the free algebra on $\omega$ generators $F(\omega)$, $[\varphi] \leq [\psi]$ iff $\vdash_{\mathbf{IPC}} \varphi \rightarrow \psi$. [2 pts]

5. Exercise 75 on p. 39: Show that the canonical frame $\mathfrak{F}$ of **IPC** is isomorphic to $\Phi(F(\omega))$. $\Phi$ is the functor mapping Heyting algebras to their corresponding Kripke frames, as described in the procedure on p. 35 (taking prime filters etc.)

   Cf. definition of $\Phi$ in the paragraph above Exercise 60. [4 pts]