Homework 5, due Tuesday 18 March, before 12.00

1. (a) Show that, if $\mathfrak{F}$ is finite and the disjoint union of $\mathfrak{F}_1$ and $\mathfrak{F}_2$, then $\mathfrak{F}_1$ is a p-morphic image of $\mathfrak{F}$ (sketched in class). [2 pts]

   (b) Show that, if $\mathfrak{F}$ is a finite tree, then, for each w in the domain of $\mathfrak{F}$, $\mathfrak{F}_w$ is a p-morphic image of $\mathfrak{F}$. [2 pts]

   (c)* Give counterexamples to (a) and (b) if $\mathfrak{F}$ is allowed to be infinite. [2 pts]

2. (a) Show that the following is valid: If $\vdash_{\mathbf{IPC}} (\varphi \to \psi) \to (\chi \vee \theta)$, then $\vdash_{\mathbf{IPC}} (\varphi \to \psi) \to \chi$ or $\vdash_{\mathbf{IPC}} (\varphi \to \psi) \to \theta$ or $\vdash_{\mathbf{IPC}} (\varphi \to \psi) \to \varphi$. [4 pts]

   (b)* Give an example such that the first two alernatives of (a) do not apply, but the last one does. [2 pts]

3. (a) Let φ contain only $\wedge, \vee$ and $\to$ but no $\neg$ and no $\bot$. Let $\mathfrak{M}$ be any Kripke-model (for the language of φ). Extend the model $\mathfrak{M}$ to $\mathfrak{M}^+$ by adding one more node x at the top above all the nodes of $\mathfrak{M}$, and making all the propositional variables of φ true in x.

   Show that, for all the nodes w in $\mathfrak{M}$ we have:

   $$\mathfrak{M}, w \models \varphi \text{ iff } \mathfrak{M}^+, w \models \varphi$$

   (satisfaction in the old and new model is the same for φ). [4 pts]

   (b) Let φ contain only $\wedge, \vee$ and $\to$ but no $\neg$ and no $\bot$. Show that $\vdash_{\mathbf{IPC}} \varphi$ iff $\vdash_{\mathbf{KC}} \varphi$. (You may use what is claimed about completeness of **KC** in class.) [2 pts]