William of Sherwood, Singular Propositions and the Hexagon of Opposition.

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Presentation

- Singular propositions.

- In traditional logic.

- In Sherwood.

- Sherwood & Hexagon of opposition

Aristotelian propositions

In Aristotelian logic, categorical propositions are divided into four basic types:

- Universal affirmative: "Every man is running".
- Universal negative: "No man is running".
- Particular affirmative: "Some man is running".
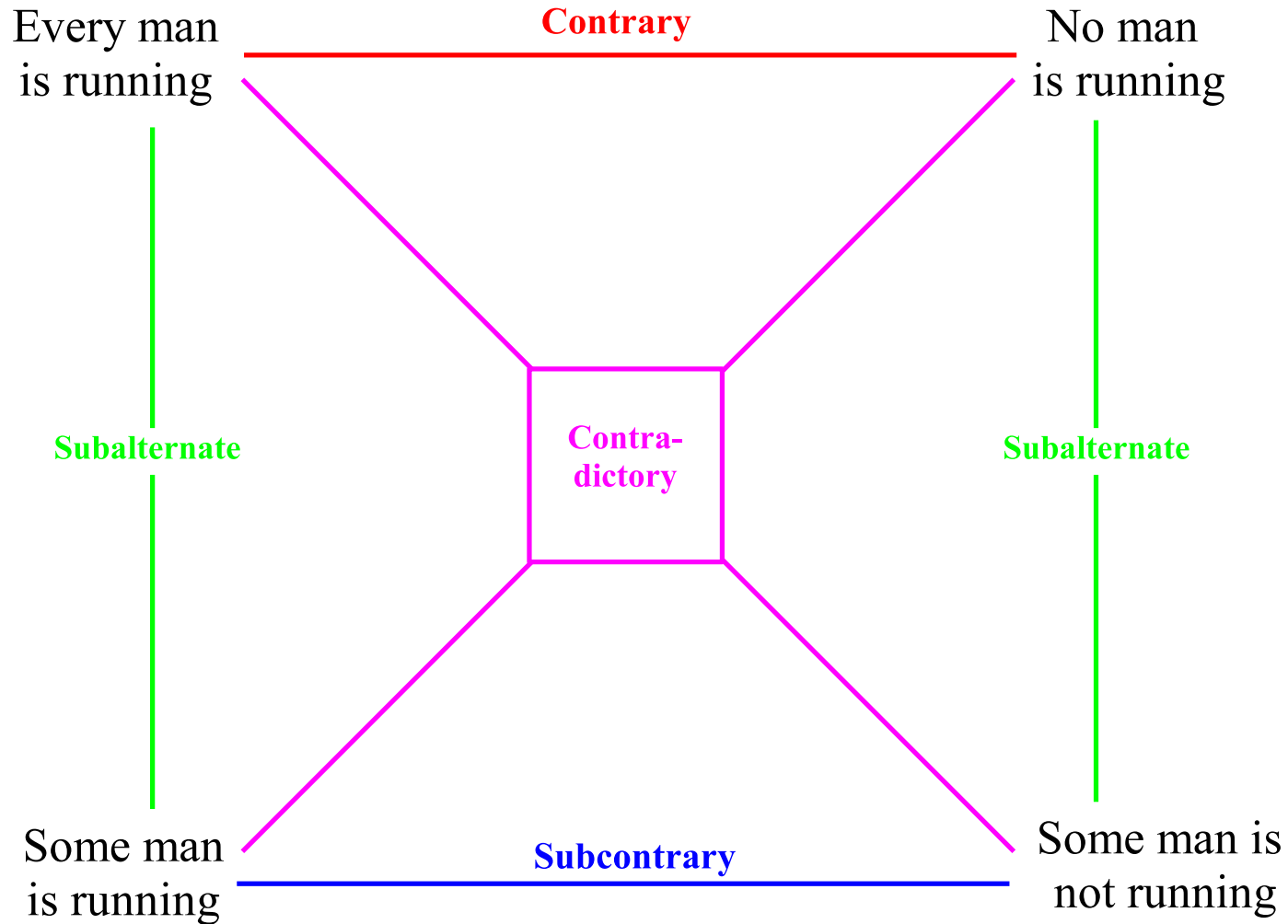- Particular negative: "Some man is not running".

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- Particular affirmative: "Some man is running".
- Particular negative: "Some man is not running".

Possible relations between two propositions: contrary, subcontrary, contradictory or subalternate.

Square of opposition

Every man is running — **Contrary** — No man is running

Subalternate

Contra-dictory

Subalternate

Some man is running — **Subcontrary** — Some man is not running

Singular propositions

What to do with **singular propositions**?

- Singular affirmative: "Socrates is running", "This man is running".

- Singular negative: "Socrates is not running". "This man is not running".
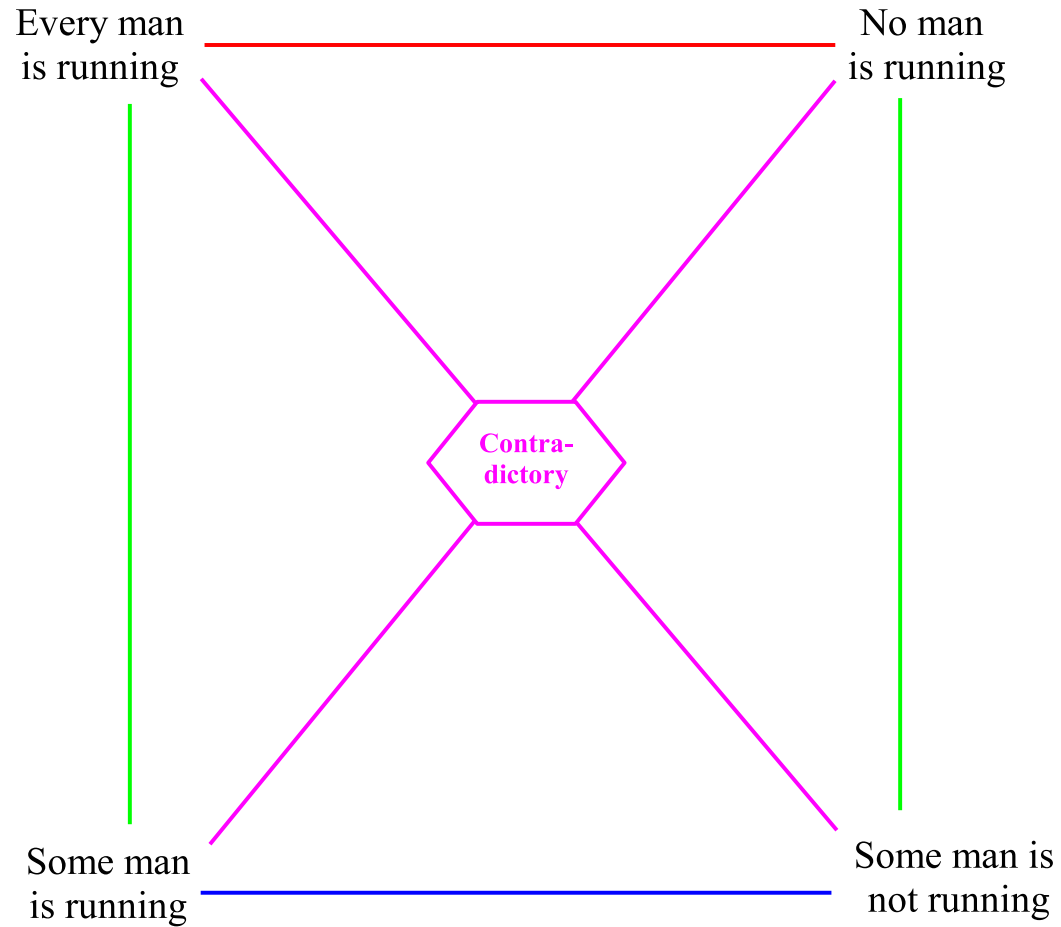
Singular propositions

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From the modern point of view:

- universal $= \forall$

- particular $= \exists$

- **singular** corresponds to a sentence with e.g. a *free variable*, or a *constant symbol*.
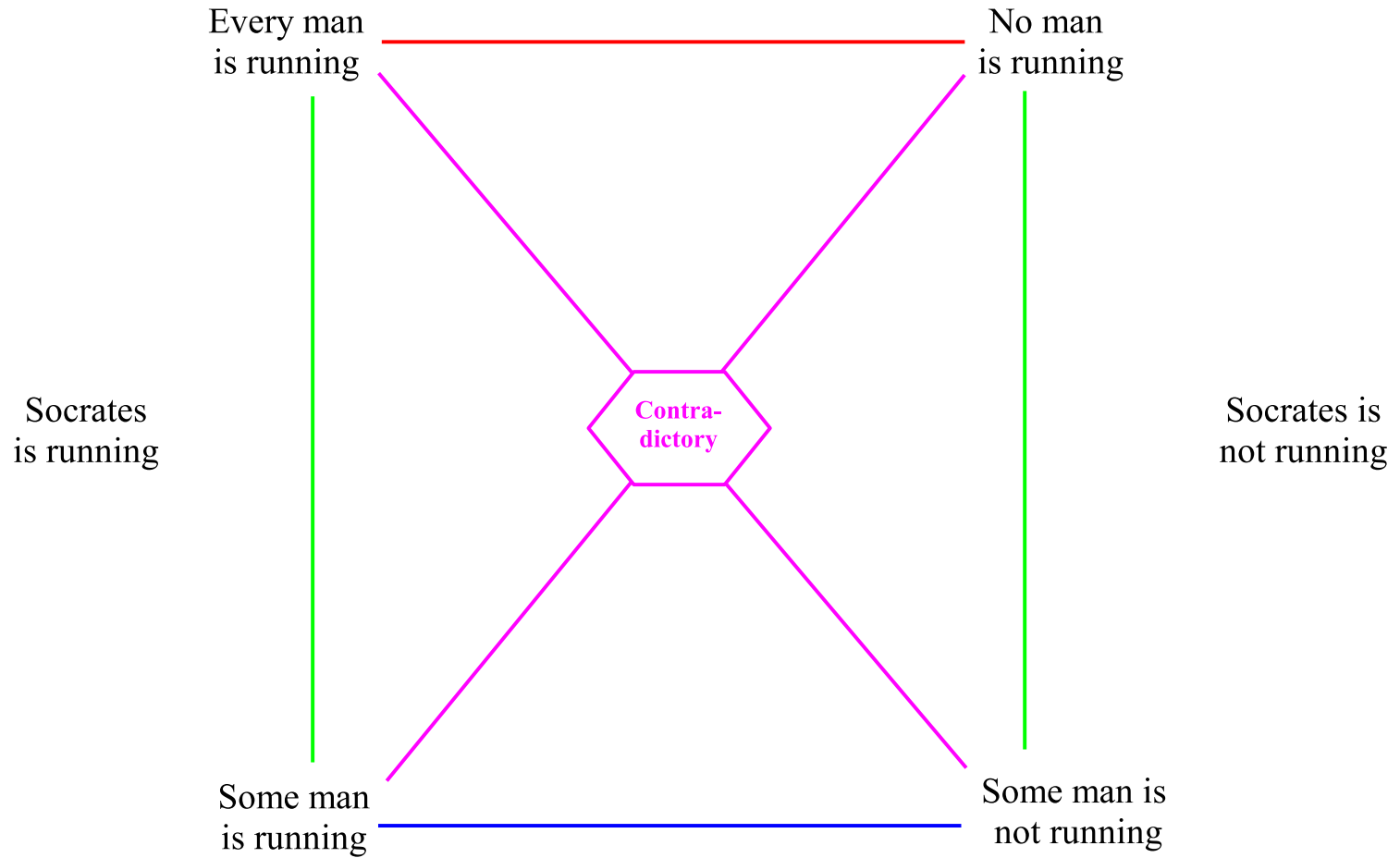
Hexagon of opposition

Every man
is running ————————————— No man
is running

**Contra-
dictory**

Some man
is running ————————————— Some man is
not running

Contrary Subcontrary Contradictory Subaltern

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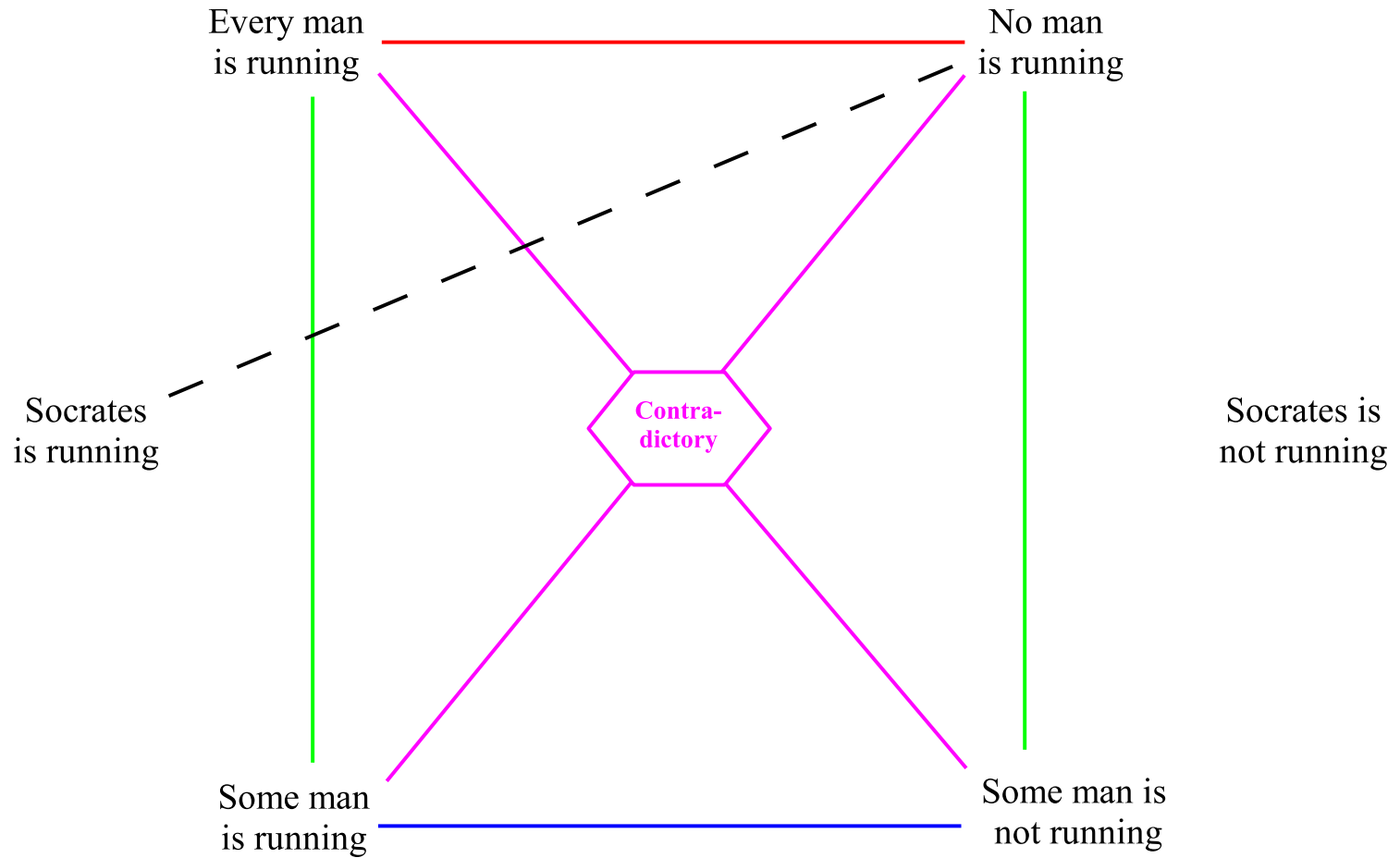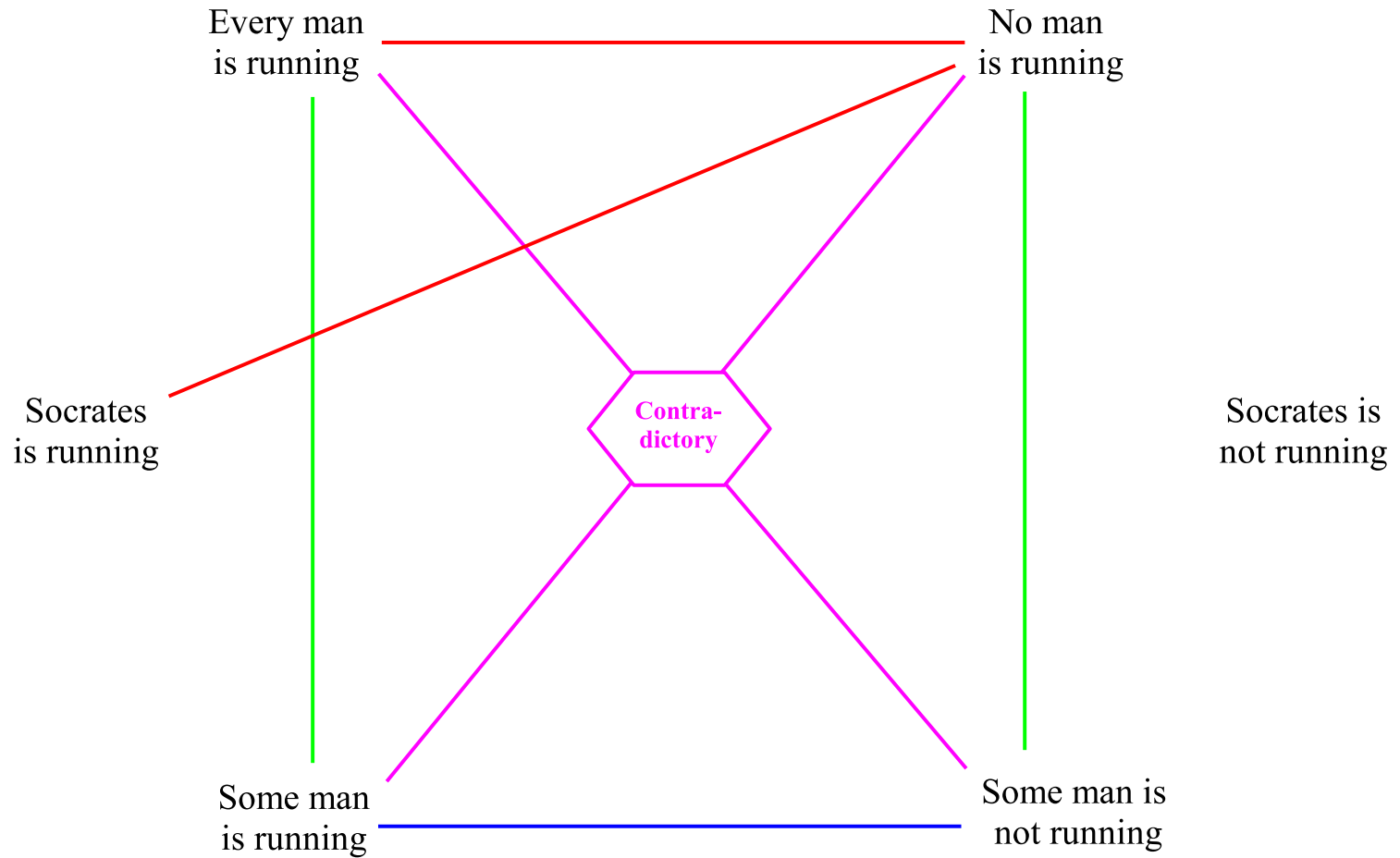
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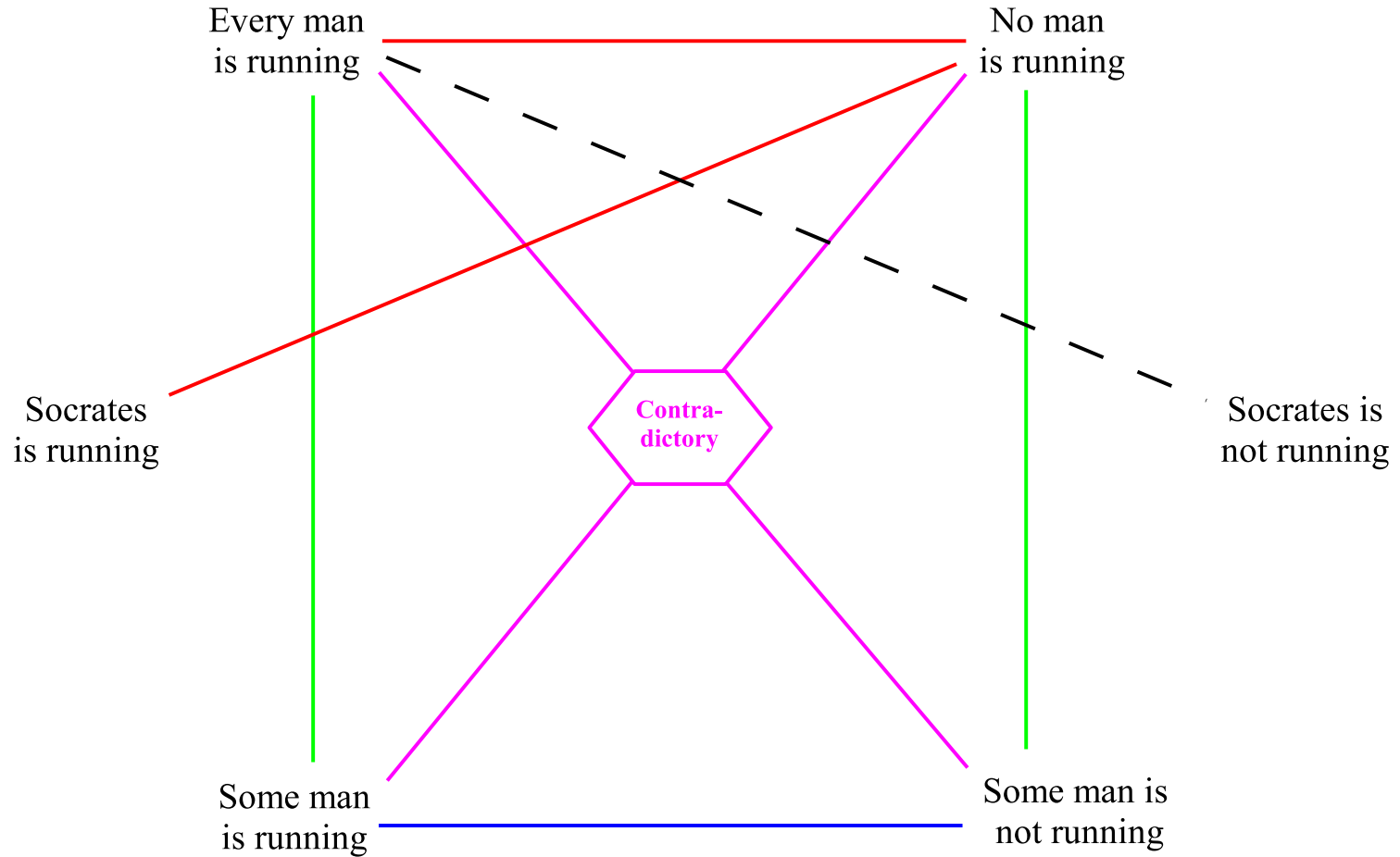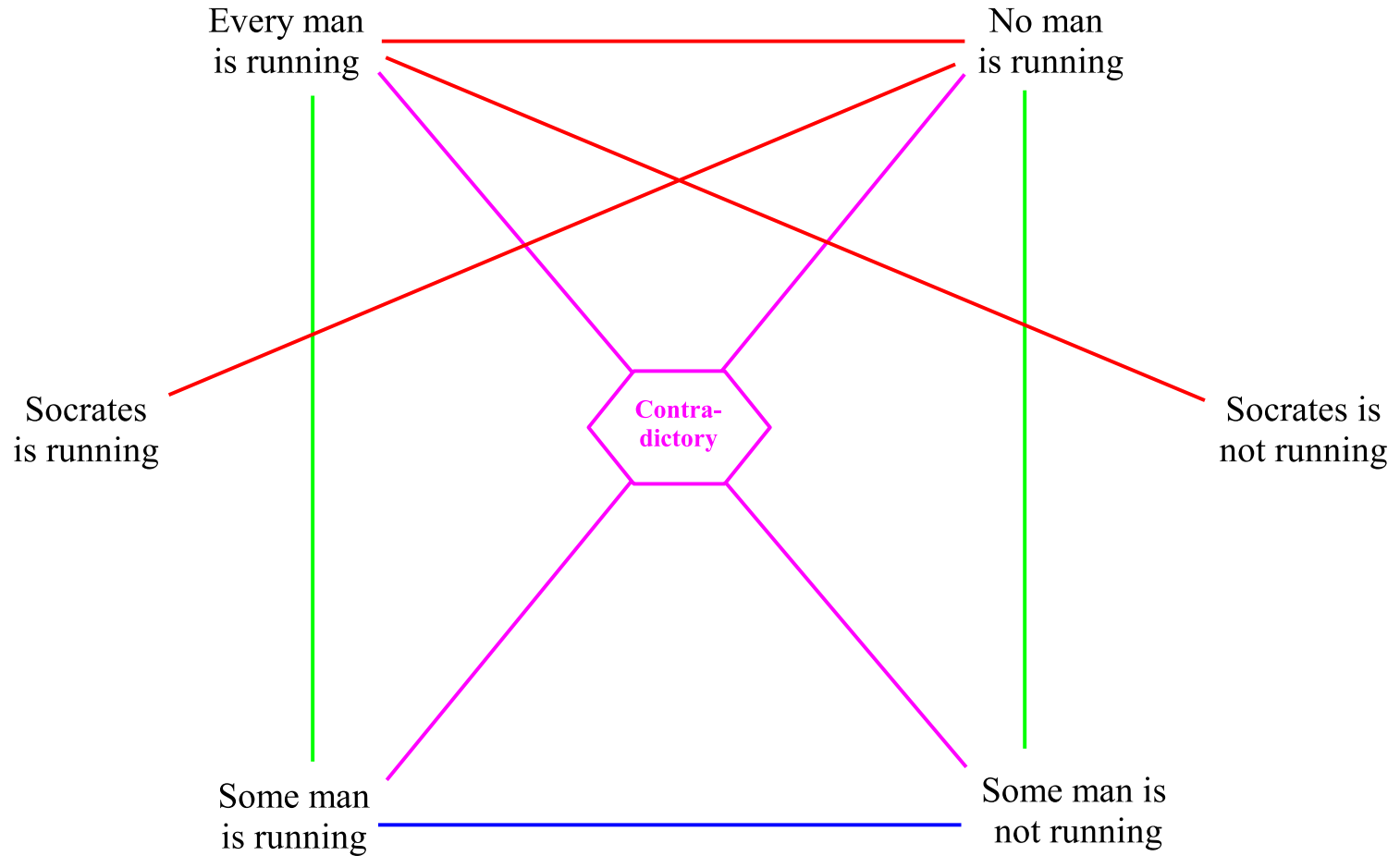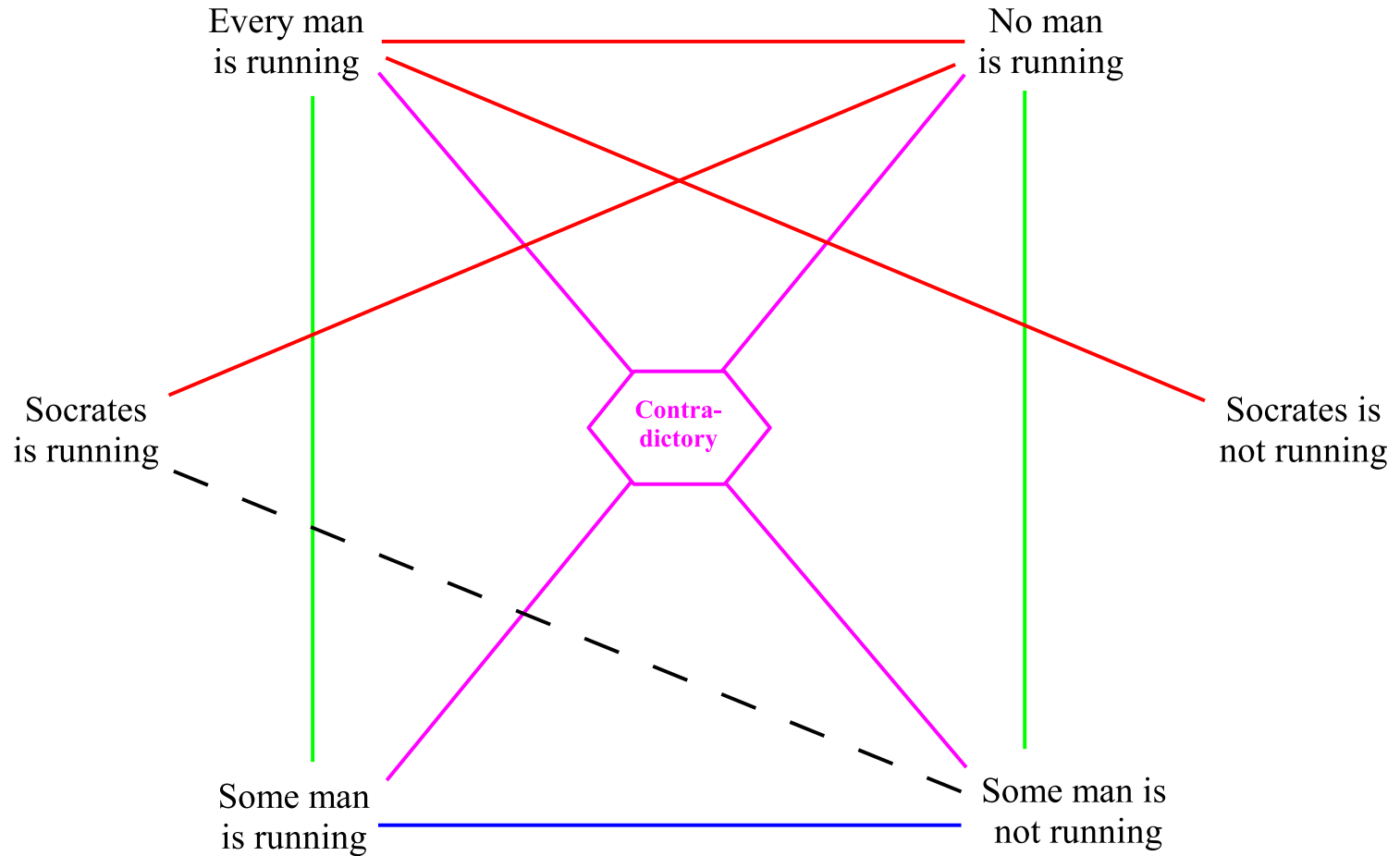Contrary   Subcontrary   Contradictory   Subaltern

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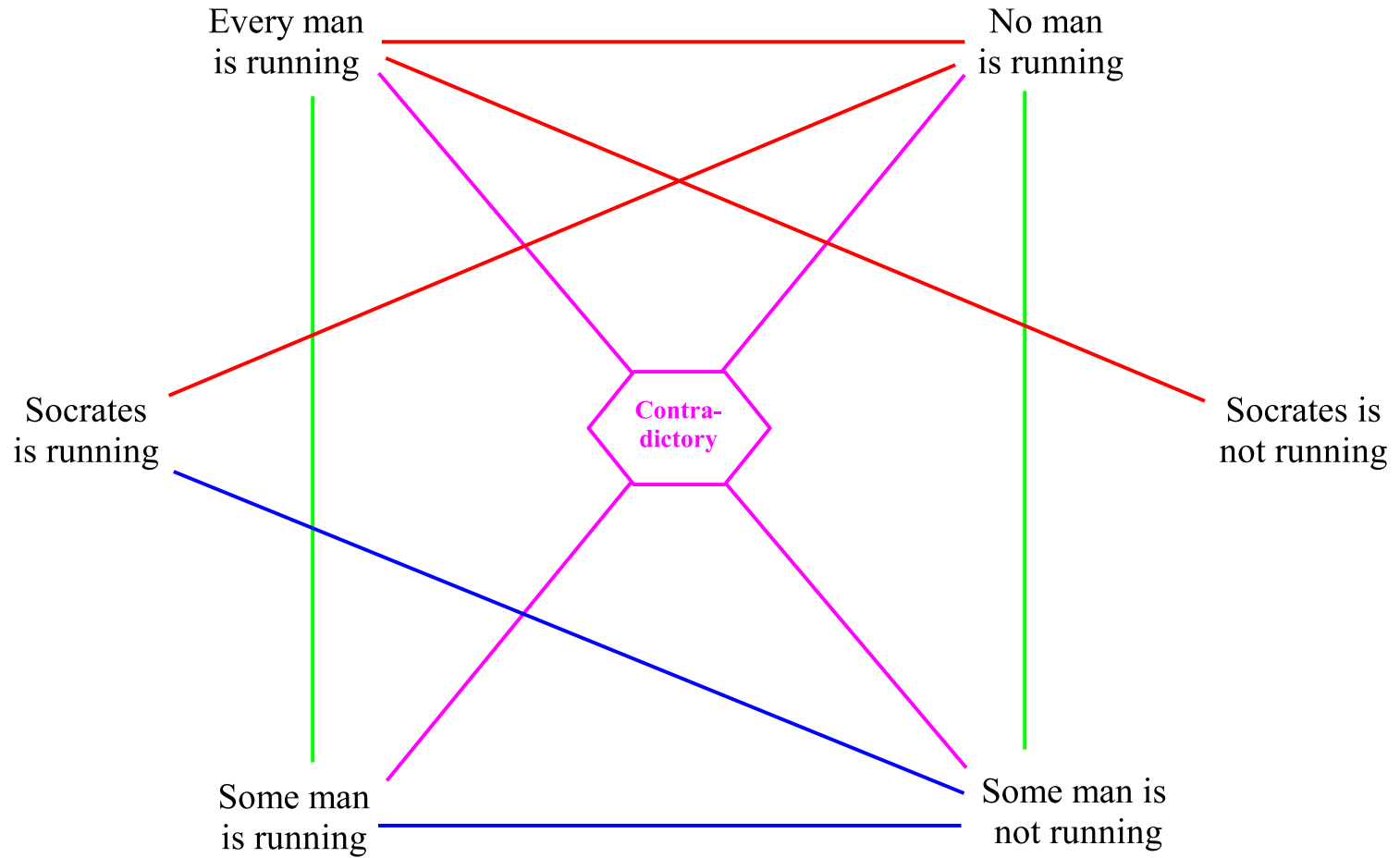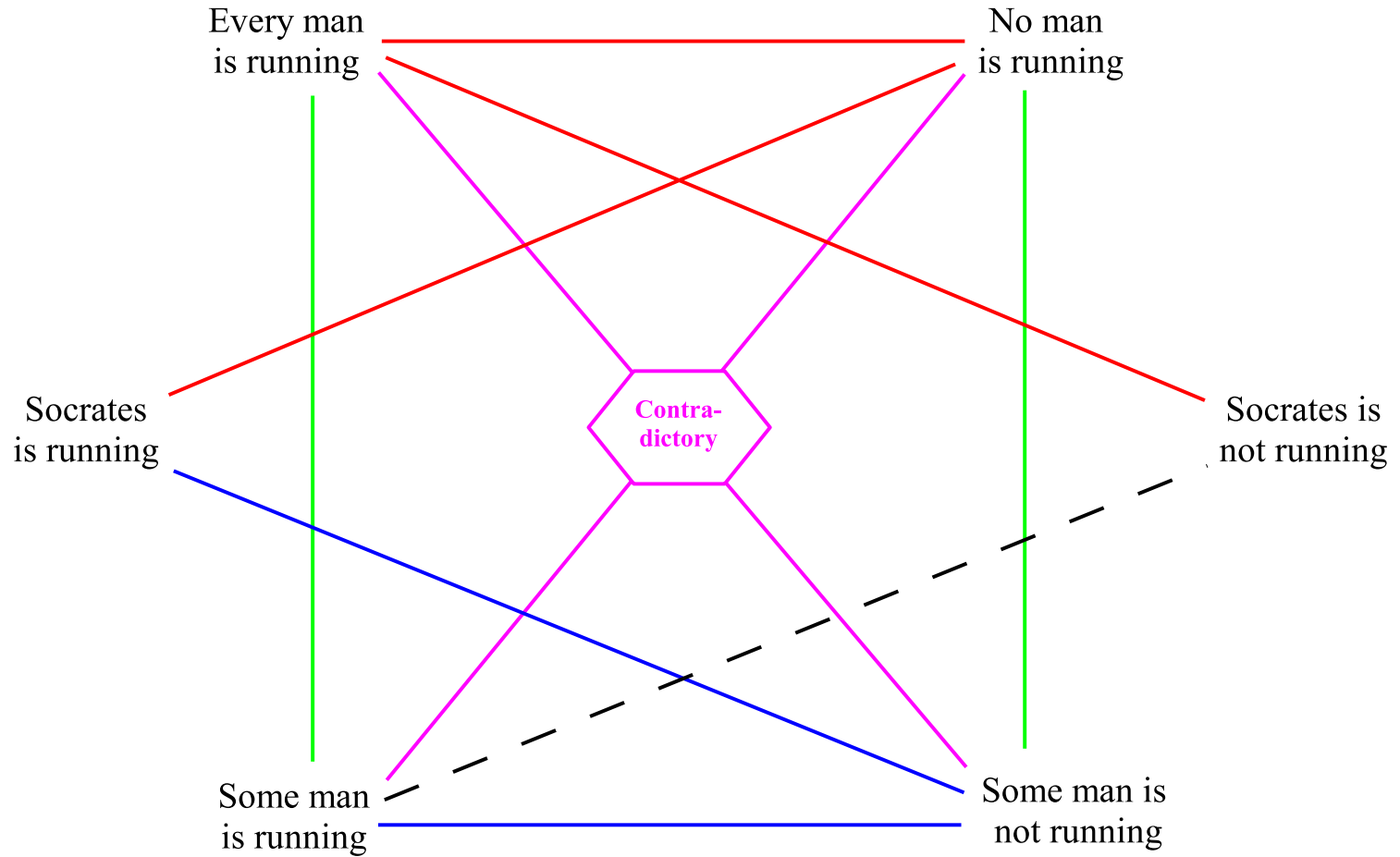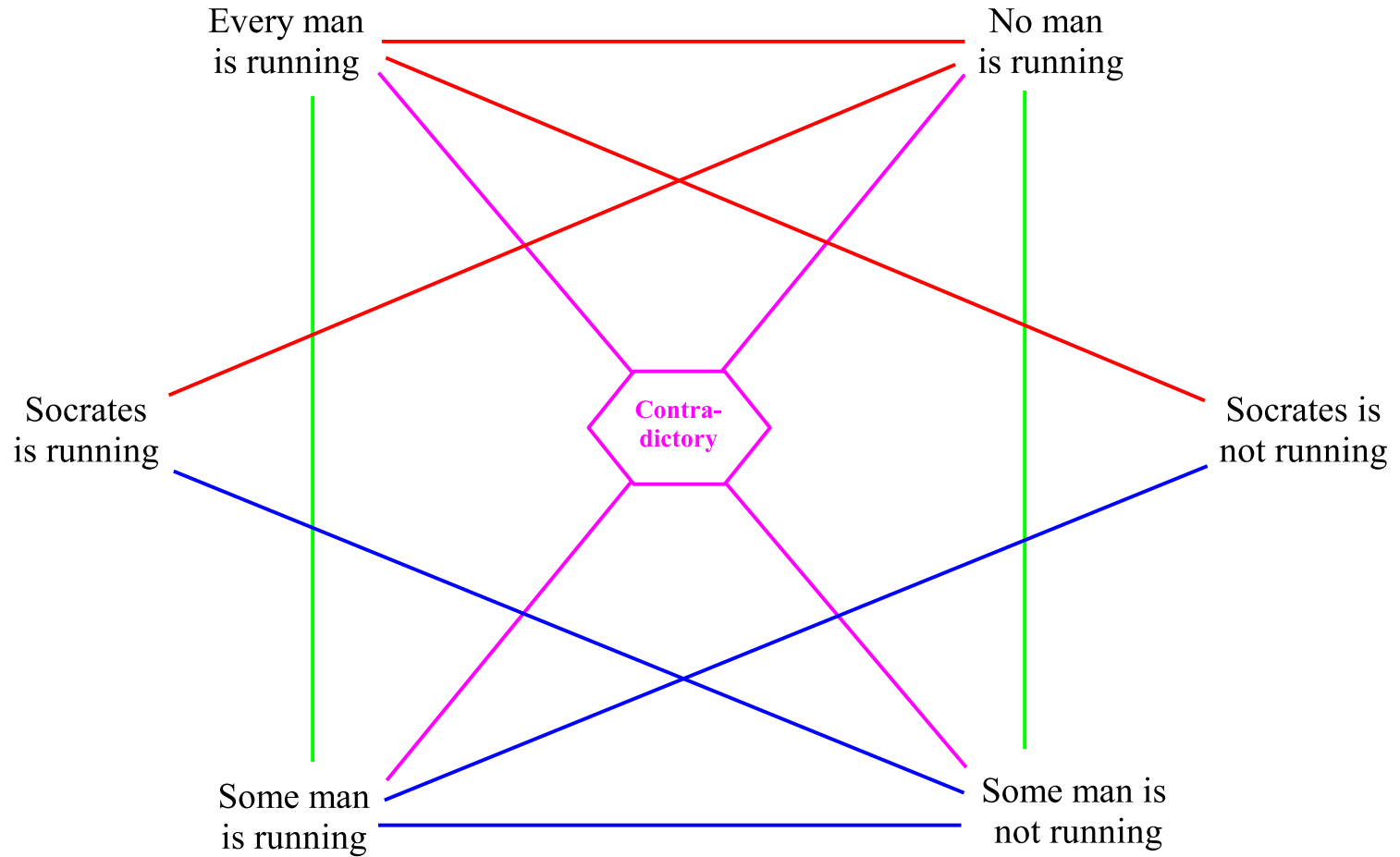
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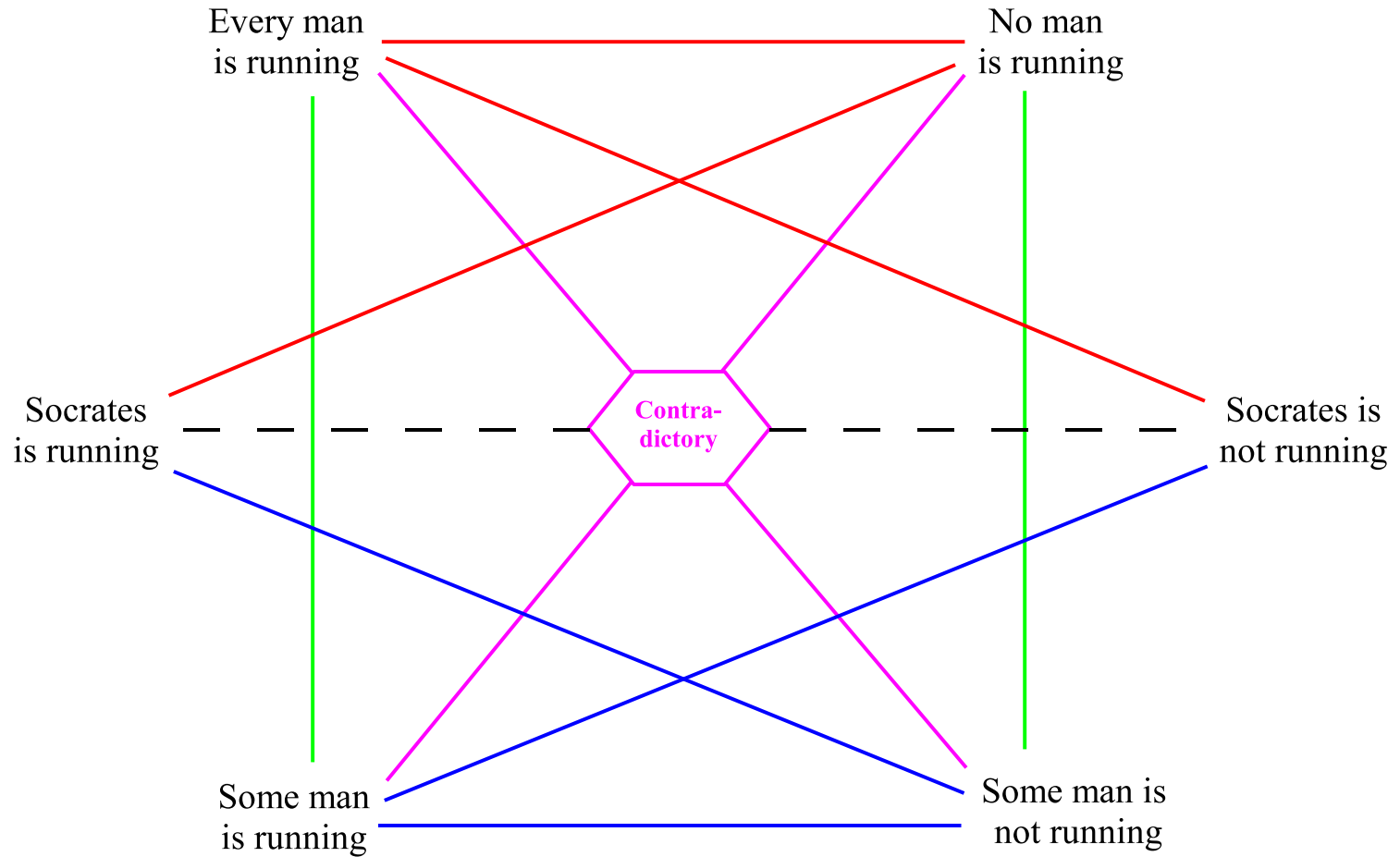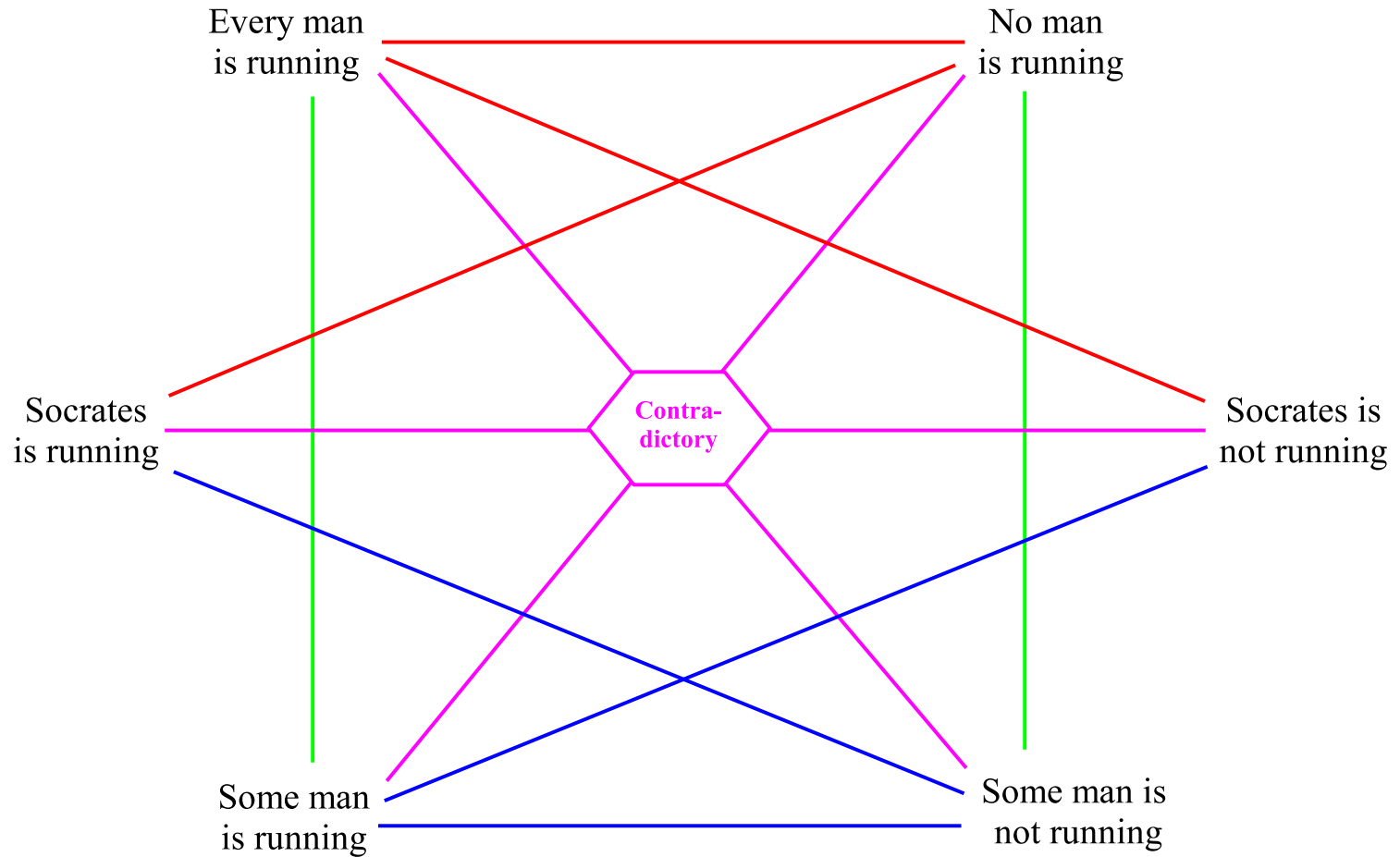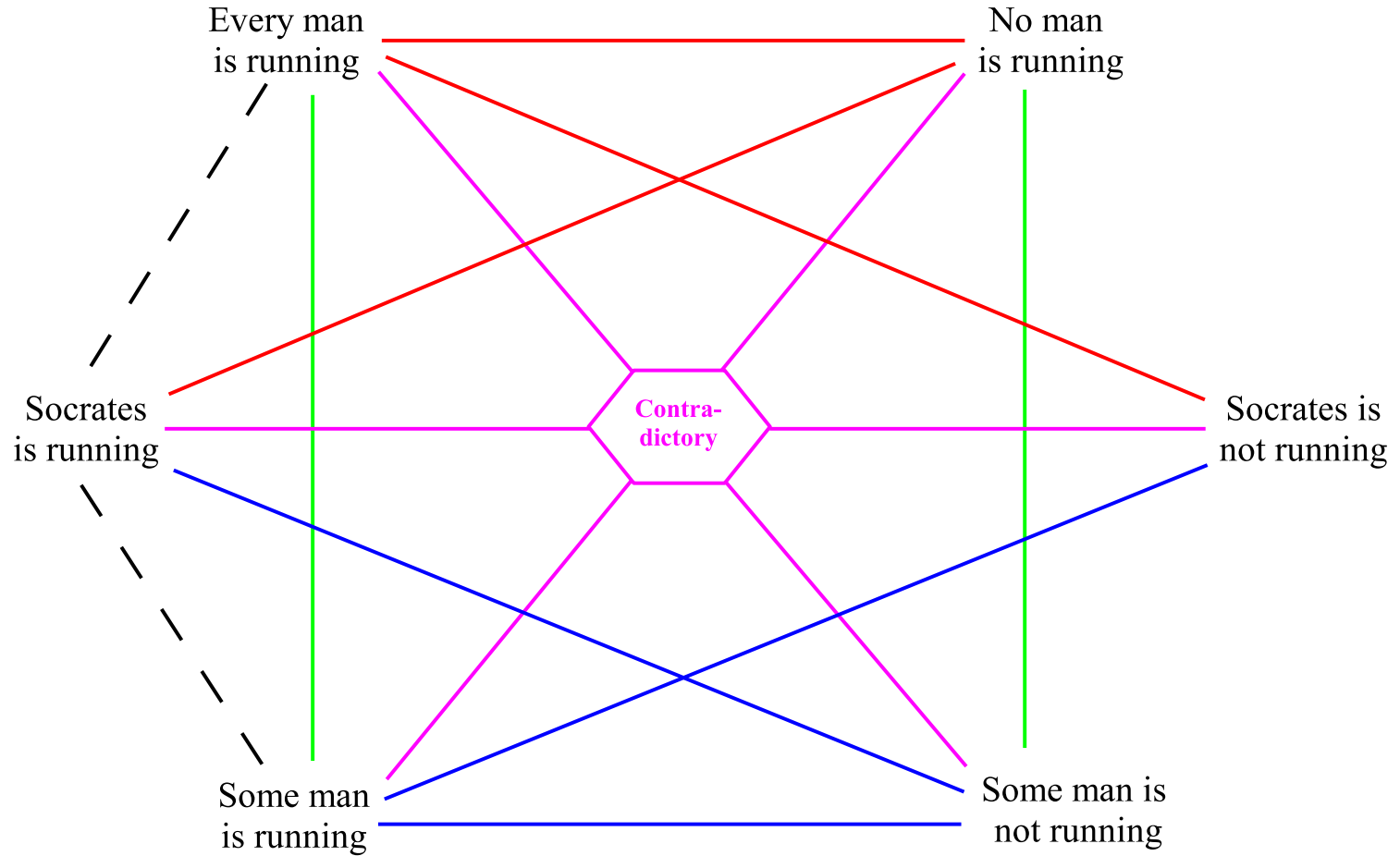
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Hexagon of opposition

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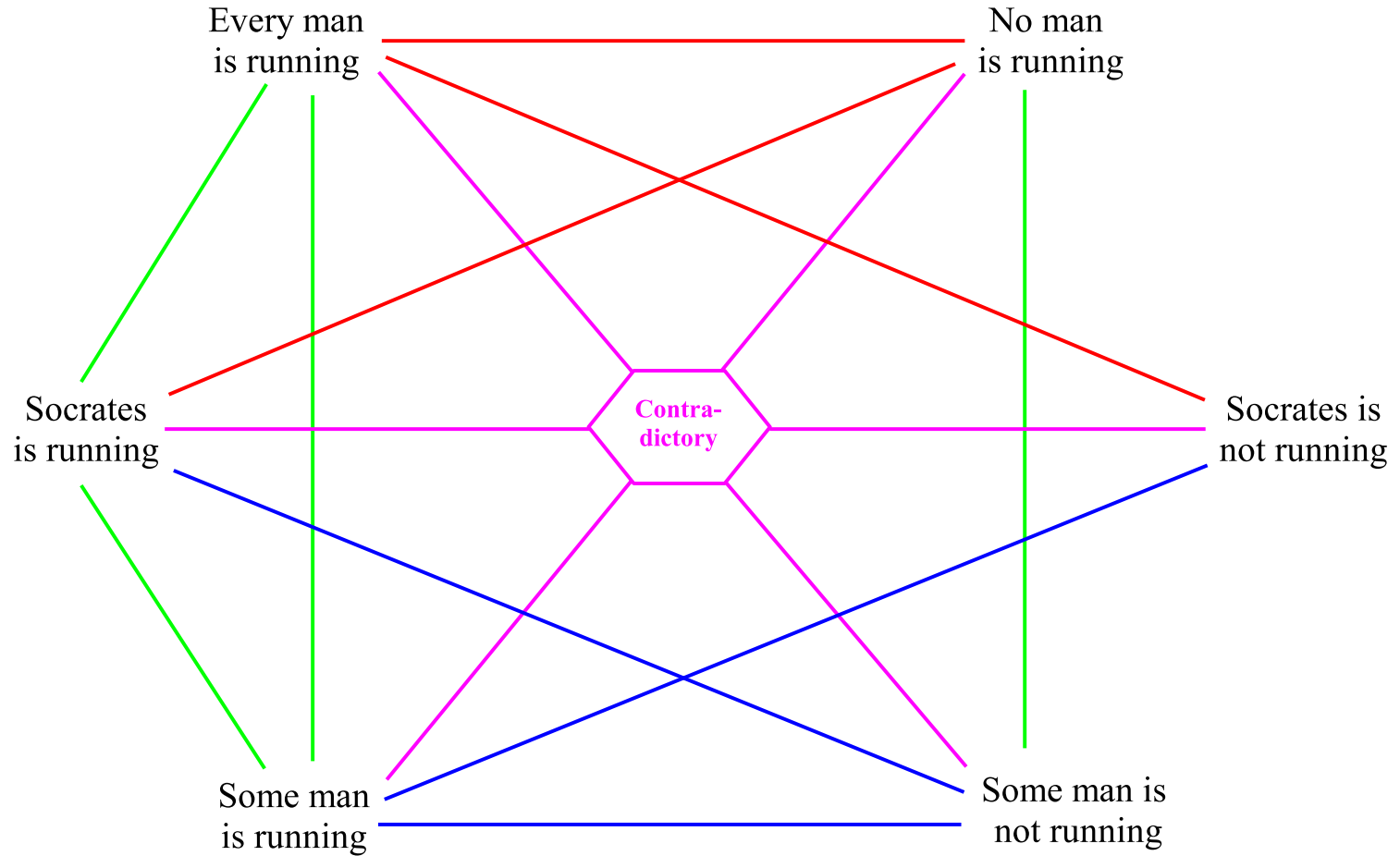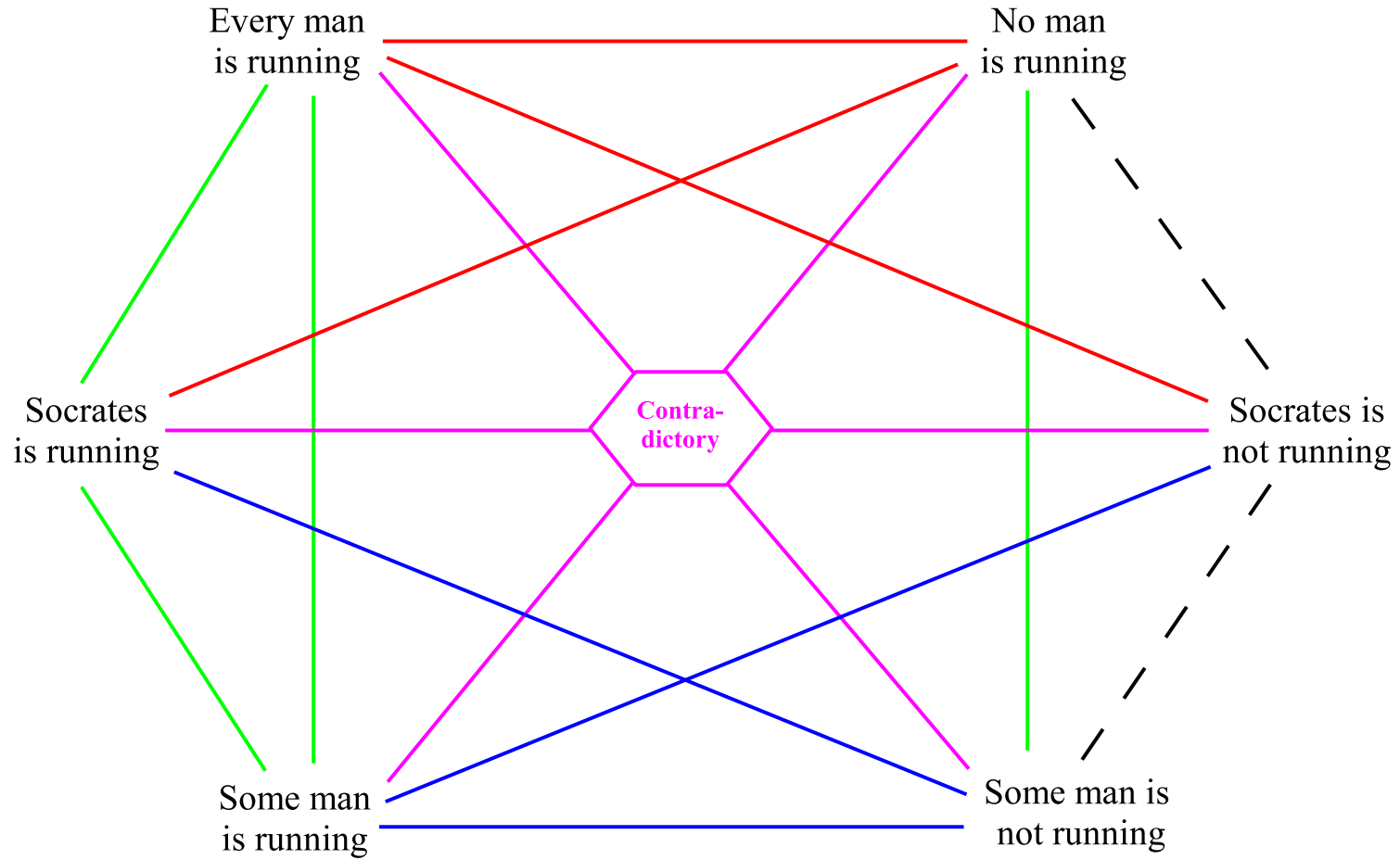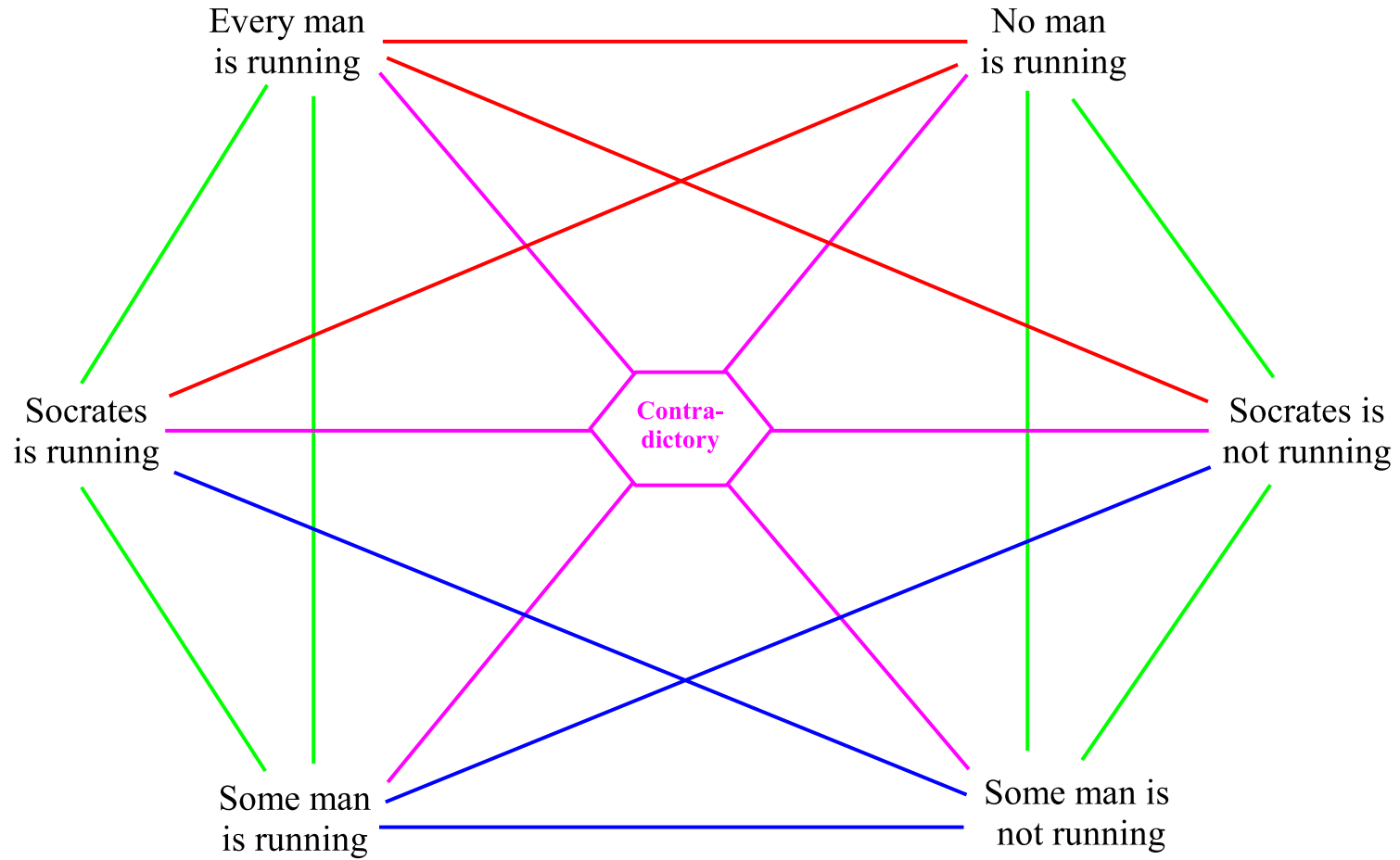
Hexagon of opposition

Every man is running — No man is running

Socrates is running — Contra-dictory — Socrates is not running

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Czeżowski

First published version:
T. Czeżowski, **"On Certain Peculiarities of Singular Propositions"** Mind 64, 1955.
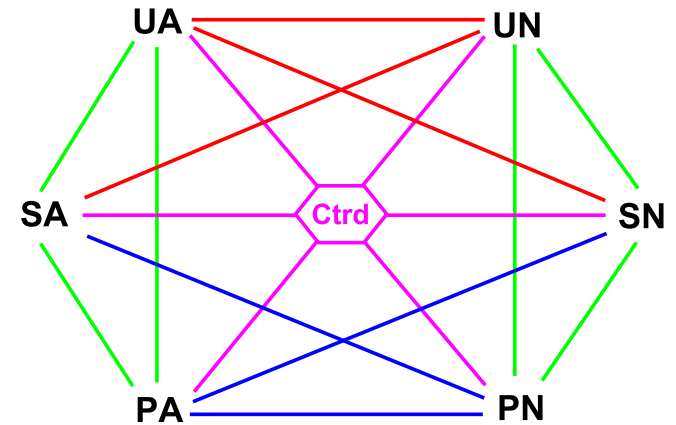
Of course, using 20th century mathematical logic, such a hexagon is easily constructed.

But what about medieval and traditional philosophy?

Traditional philosophy

Doctrine of traditional logic: there are only **two** types of quantities: universal and particular.

Debate: is a singular proposition particular or universal? Most common view: universal!

Arguments

Some arguments:

- In a singular proposition, predication is of one individual, which is even less than in a particular.

- In a singular propositions, predication is of the whole of the subject, just like in a universal.

  "This man is running" — is the subject term "man" or "this man"?

Examples

- Aristotle: mentions very briefly that singular affirmative and singular negative are mutually contradictory.

  "…one must be true, the other false. This also holds good of propositions with singular terms for their subjects, as 'Socrates is white' and 'not white'.

  (*On Interpretation* Ch. 7, 17b)

  But not treated in *Prior Analytics*.

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- John Wallis, 1631: seems originator of the idea to regard singulars as universals.

  "A singular proposition, in a syllogistic disposition, always has universal force"

  (*Institutio Logicae, appendix*)

Examples

- Arnauld & Nicole, 1662 (Port Royal Logic):

"Although singular propositions differ from universals in not having a common subject, they should nevertheless be classified with them rather than with particulars, because they have a singular subject which is necessarily taken through its entire extension."

(*La logique ou l'art de penser*, Part II, Ch.3)

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- ## Euler, 1761:

  "Certain authors insist, that a singular proposition must be ranked in the class of particulars; . . . However well founded this reasoning may appear, it cannot be admitted . . . It is clearly evident . . . that a singular proposition must be considered as universal".

  (*Lettres à une Princess d'Allemagne*, CVII)

Examples

- Leibniz:

"It should be noted that (as far as the form is concerned) singular sentences are put with the universals. For, although it is true that there was only one Apostle Peter, one can nevertheless say that whoever has been the Apostle Peter has denied his master."

(*Nouveaux Essais sur l'entendement humain*, IV. XVII. 8)

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(*Nouveaux Essais sur l'entendement humain*, IV. XVII. 8)

"How is it that opposition is valid in the case of singular propositions—e.g. 'The Apostle Peter is a soldier' and 'The Apostle Peter is not a soldier'—since elsewhere a universal affirmative and a particular negative are opposed? Should we say that a singular proposition is equivalent to a particular and to a universal proposition? Yes, we should."
(*On some logical difficulties*)

Examples

Even in the early 20th century:

- Keynes, *Studies and Exercises in Formal Logic*, 1906:
  "Singular propositions may be regarded as forming a sub-class of universals, since in every singular proposition the affirmation or denial is of the *whole* of the subject."

  (Part II, Ch. 2)

Opposition of singular proposition

How was the **opposition** of singular propositions analyzed?
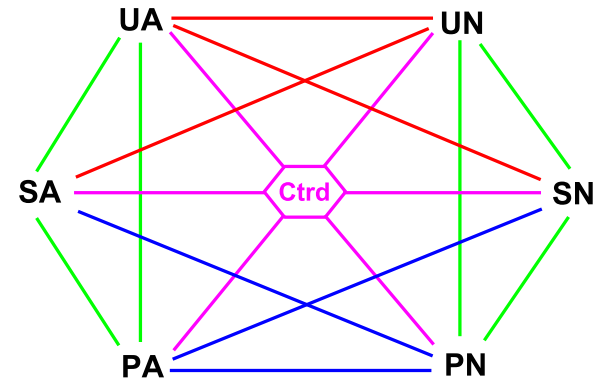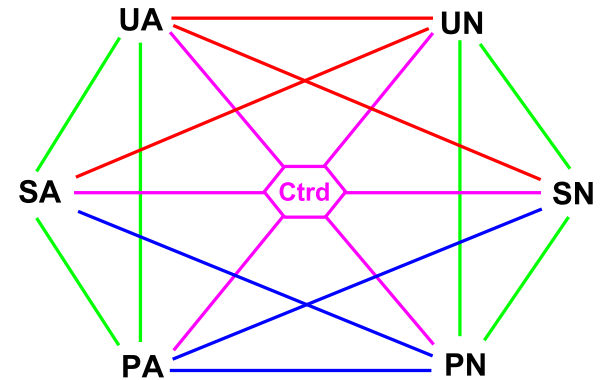
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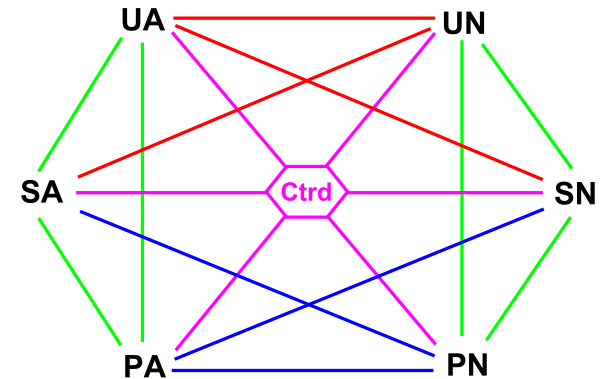
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Opposition of singular proposition

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- Keynes: "Taking the proposition *Socrates is wise*, its contradictory is *Socrates is not wise*; and so long as we keep to the same terms, we cannot go beyond this simple denial. The proposition has, therefore, no formal contrary."

Back to Czeżowski

1955: Czeżowski presents the hexagon of opposition, and the corresponding relations as a "new discovery":
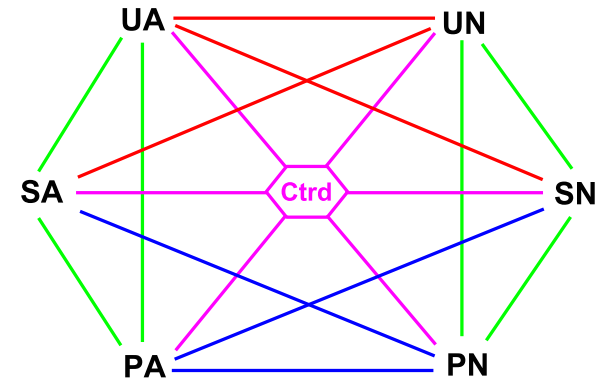
"...a distinction ought to be made between singular and universal propositions."

"Opposition relations among the six propositions thus distinguished will be represented on a hexagon, analogously to the logical square."

"...it has been inferred that there is no proposition that might properly be the contrary of a singular proposition. This inference is wrong."

"Contrary to the belief quoted above, both singular propositions do have their contraries—namely, universal propositions. But at the same time they are placed in a relation of subcontrariety to particular propositions."
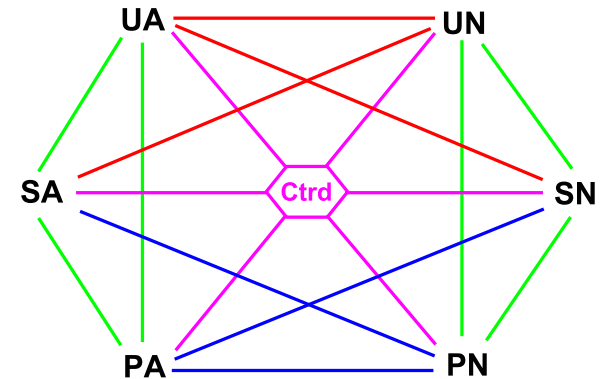
Sherwood

Let's go back some 800 years.
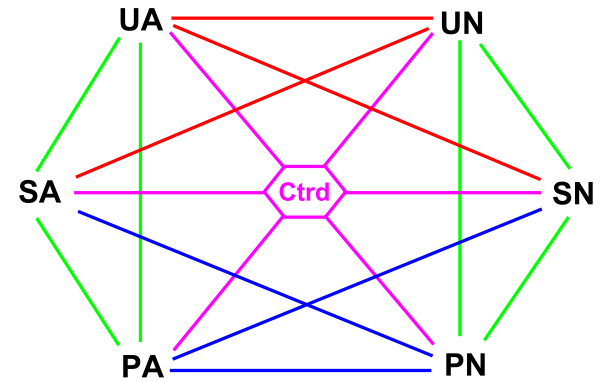
Sherwood

Let's go back some 800 years.

William of Sherwood:

- English philosopher and logician, 1190–1249.

- Not much known of his life.

- Main works: *Introductiones in Logicam* and *Syncategoremata*.

The *Introductions* have survived in just **one** manuscript, probably from late 13th or early 14th century.

Sherwood

"If two statements are singular and of different quality they are not subcontraries but, in accordance with the theory, contradictories – e.g. 'Socrates is running', 'Socrates is not running'.

Note, moreover, that **a universal affirmative and a singular negative, as well as a universal negative and a singular affirmative, are mutually contrary** (at least as far as the law goes) **because they can be false at the same time and cannot be true at the same time.** Suppose that Socrates is running and no one else; in that case these statements are false: 'every man is running', 'Socrates is not running.' Again, suppose that Socrates is not running but everyone except him [is running]; then these statements are false: 'no man is running,' 'Socrates is running'."

"Universalis affirmativa et singularis negativa et etiam universalis negativa et singularis affirmativa contrariantur ad minus quantum ad legem, quia possunt simul esse false et non simul vere."
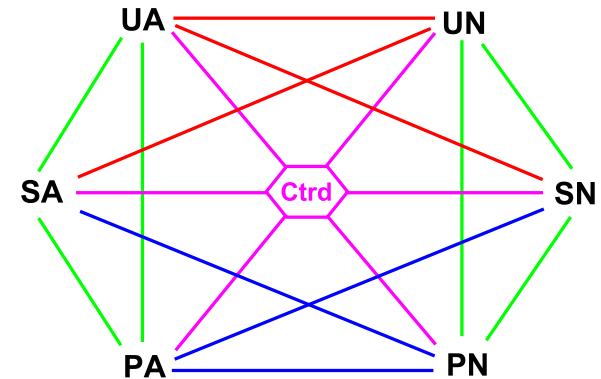
Sherwood

Sherwood, in the early 13th century, had **essentially** the same idea as Czeżwoski in 1955.

Sherwood

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Why only Sherwood?

- Particular passage overlooked by later logicians?

- Misunderstood?

- Not deemed important enough?

- Didn't fit the dichotomy: universal vs. particular?

- Philosophers focused on other, more "philosophical" nature of singulars, rather than just the opposition relation they form with universals and particulars?

The hexagon

Having established Sherwood's accomplishment in this area, we ask an even more provocative question:
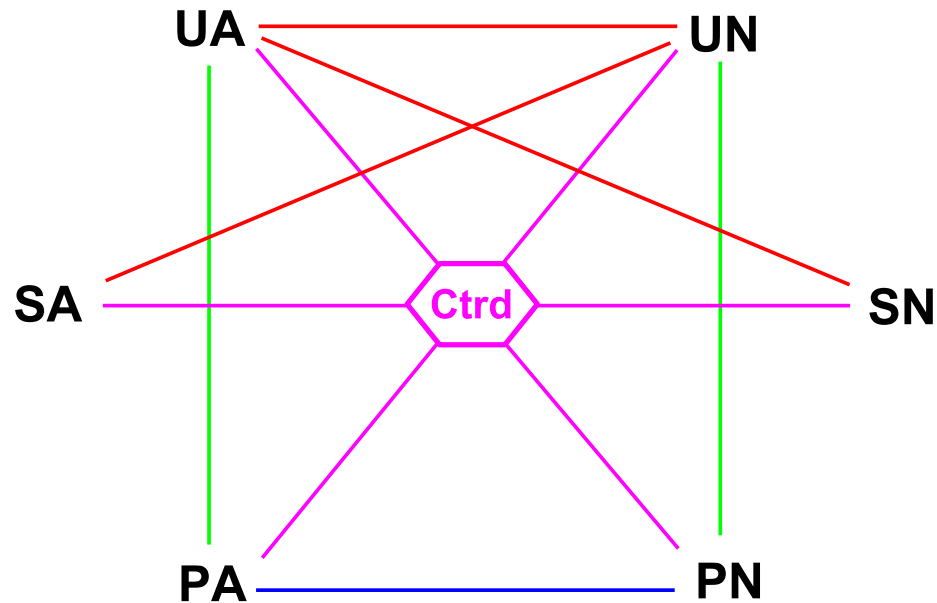
The hexagon

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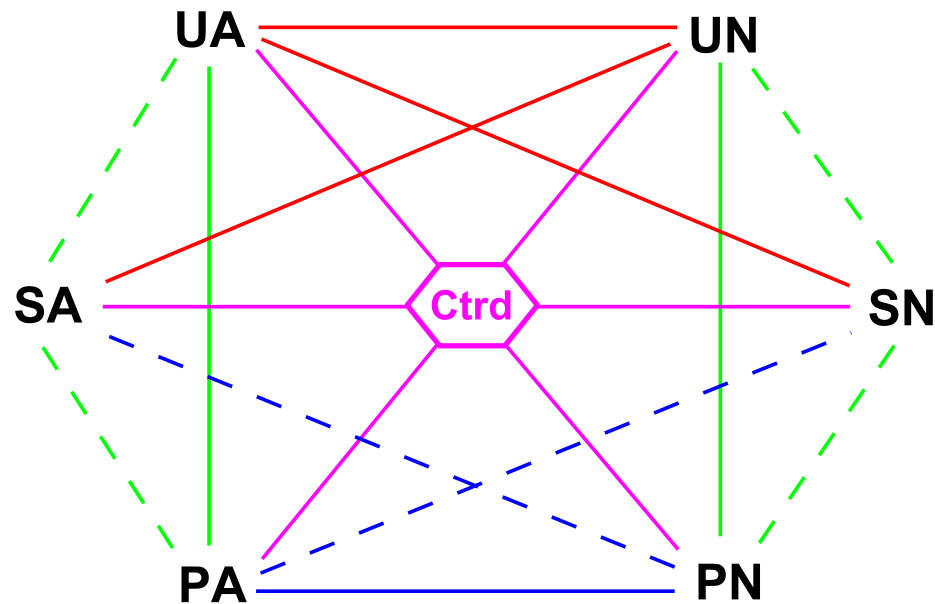Could Sherwood even have conceived the Hexagon of Opposition **as a diagram**?

The hexagon

From all the relations in our hexagon, the following are explicitly mentioned by Sherwood:

The hexagon

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The others can easily be inferred.

The hexagon

"[Passage quoted above]. This, then, is the division of statements arising from the arrangements or relation of one statement with another—viz. some are contraries, some subcontraries, some subalterns, and some contradictories, **as in the figure below.**"
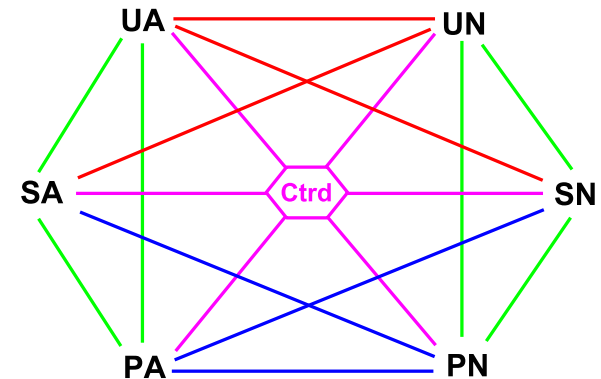
"*Est igitur hec divisio enuntiationis, que accidit ei in ordinatione ad alterum sive secundum comparationem scilicet quod quedam sunt contrarie, quedam subcontrarie, quedam subalterne, quedam contradictorie, ut in subiecta figura.*"

- Which figure does this refer to?

The hexagon

Since "as in the figure below" follows directly upon the description of the relations, we would expect the figure to be some kind of hexagon.

But in the (only surviving) manuscript, it is the **standard square**.

The hexagon

Two explanations:

1. Sherwood did **not** draw anything like the hexagon, leaving out singular propositions and their opposition relations.

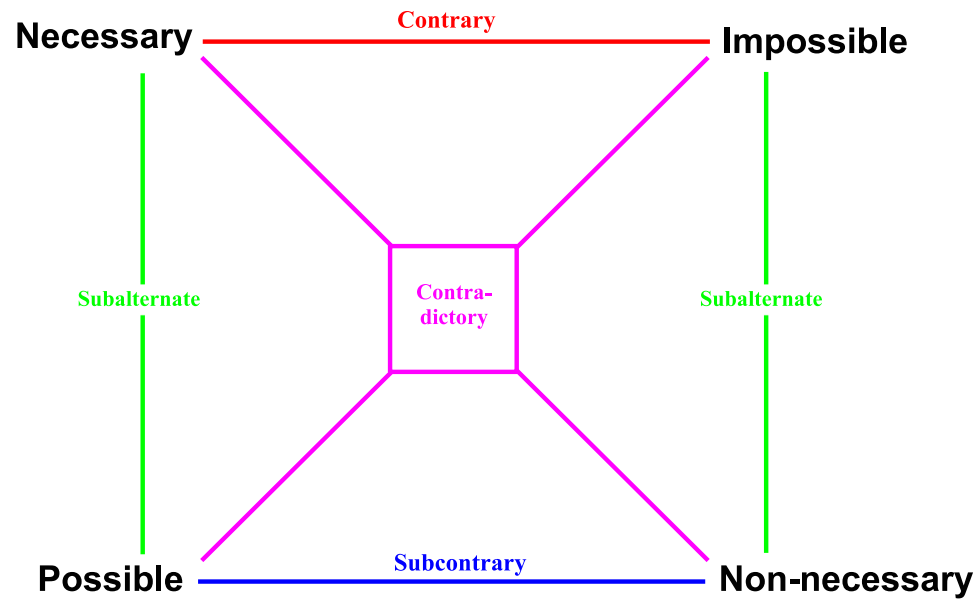2. He **did** draw something like the hexagon, but it is not preserved in the manuscript.

Evidence for the hexagon

To support the second claim, let's look at another case where something strange is going on with Sherwood's diagrams.
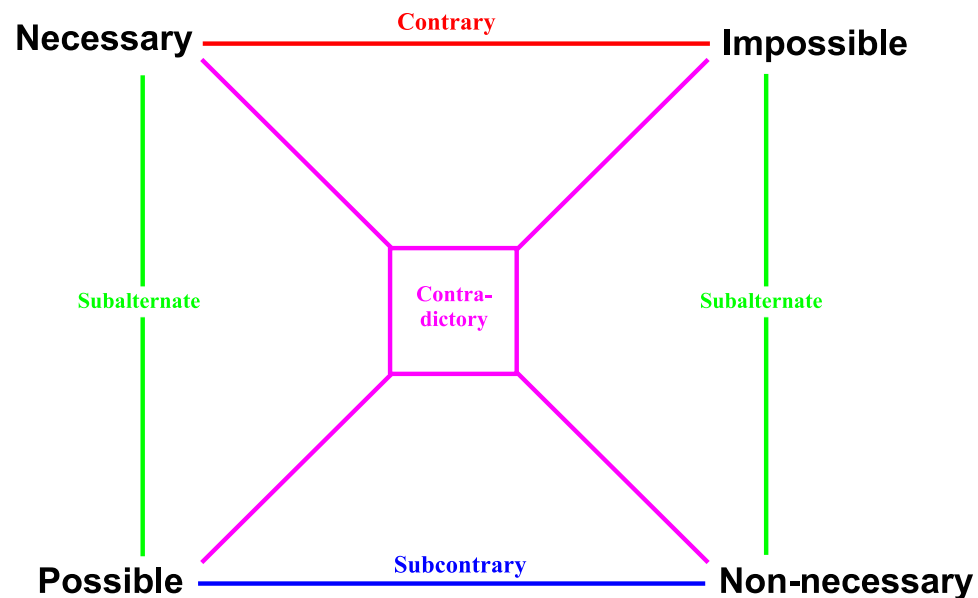
Several chapter further in the *Introductions*.

- Discusses four modalities: necessary, possible, unnecessary and impossible.

- Says that these, likewise, are related through contrariety, subcontrariety, contradiction and subalternation.

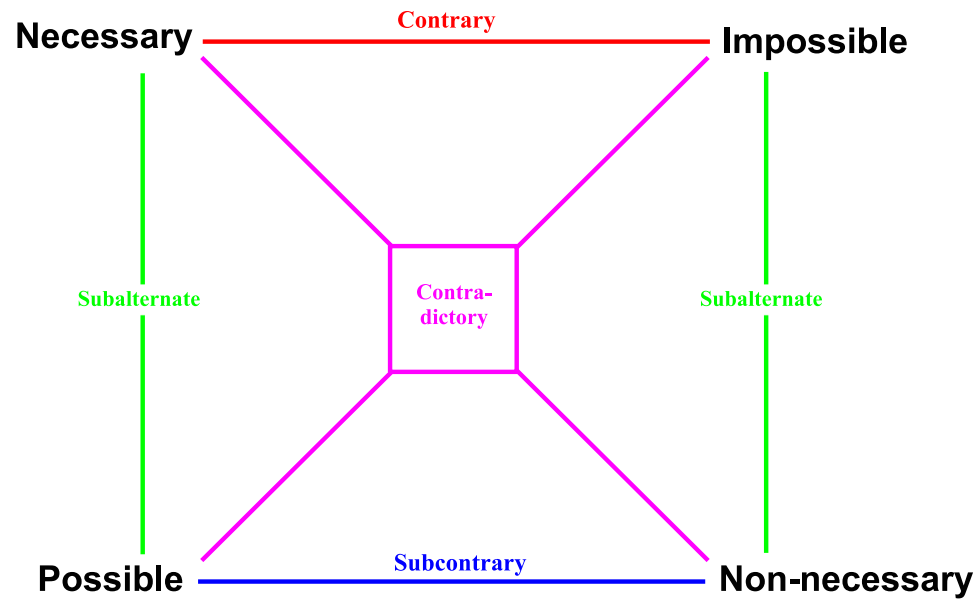- Presents the modal square of opposition.

Modal Square of Opposition

Necessary — *Contrary* — Impossible

Subalternate   *Contra-dictory*   *Subalternate*

Possible — *Subcontrary* — Non-necessary

Modal Square of Opposition

Necessary —— Contrary —— Impossible

Subalternate · Contra-dictory · Subalternate

Possible —— Subcontrary —— Non-necessary

"All these relations also appear in the accompanying figure. The figure could be arranged differently, however, so that the contrary series could be put in the first, or upper, line, and the subcontraries in the lower. But [the arrangement as given] coincides more closely with Aristotle's"
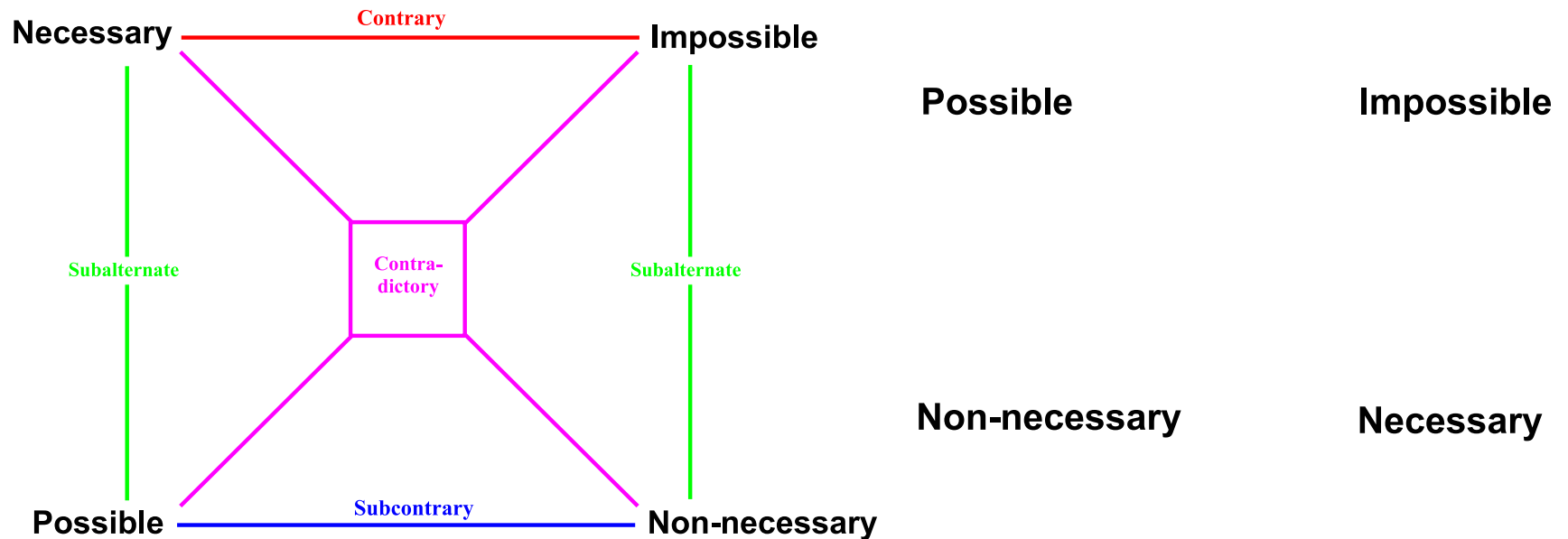
"Et hec omnia patent in figura. Posset tamen figura aliter ordinari, ut ordines contrarii ponerentur in prima linea, que est superior et subcontrarii in inferiori. Sed iste magis competit modo Aristotelis."

Modal Square of Opposition

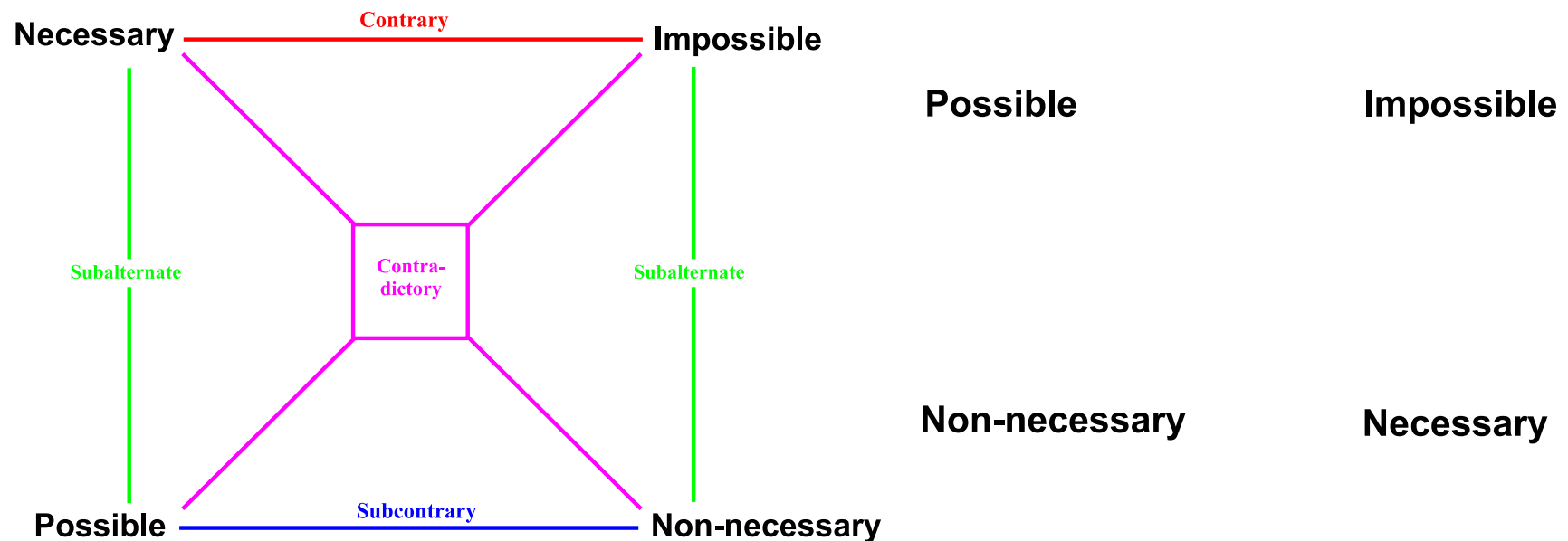How did Aristotle arrange the modalities?

Modal Square of Opposition

Necessary —— *Contrary* —— **Impossible**

Possible   **Impossible**

Subalternate   Contra-dictory   *Subalternate*

Non-necessary   **Necessary**

Possible —— *Subcontrary* —— **Non-necessary**

How did Aristotle arrange the modalities?

On Interpretation, Ch. 13 (22a)

Modal Square of Opposition

How did Aristotle arrange the modalities?

On Interpretation, Ch. 13 (22a)

So Sherwood basically says: "the figure **could** be arranged differently, but I am giving Aristotle's arrangement".

But that is not the diagram found in the manuscript!

Sherwood's diagrams

So?

- Later students and scribes **changed** Sherwood's intended Modal Square, following his suggestion.

- It seems that we cannot assume that Sherwood's original diagrams are preserved in the current manuscript.

- If so, could it be that Sherwood intended some diagram like the hexagon of opposition to be there?

Conclusion

Whatever the case with the diagrams, **Sherwood** should deserve credit for a theory of singular propositions which seems superior to 17th, 18th and even 19th century theories.