Exercises for Geometric Group Theory

Sheet 9

Exercise 1: Let Γ be a connected graph and let G be a group that acts without inversion on Γ. Show that for every subtree T of Γ/G there exists a subtree T_Γ of Γ such that $\rho|_{T_\Gamma}\colon T_\Gamma \to T$ is an isomorphisms, where ρ is the canonical projection $\Gamma \to \Gamma/G$.

Exercise 2: Complete the proof of Proposition 4.2.13, that is, show that the graph T_Γ as constructed in that proof satisfies the properties as claimed in the proposition.

Exercise 3: Define the fundamental group of graphs with respect to spanning trees (similar to the first definition of the fundamental group of graphs of groups) and show that you definition is equivalent to the definition from the lecture (and thus show that the group does not depend on the choice of the spanning tree).

Exercise 4*: Let $\mathbb{G} := (\mathcal{G}, \Gamma, \Lambda)$ be a graph of groups, where Γ is finite, and let $e \in E(\Gamma)$.

(1) If $\Gamma - e$ is not connected, then show that $\pi_1(\mathbb{G}) \cong A *_{G_e} B$ for some groups A and B.

(2) If $\Gamma - e$ is connected, then show that $\pi_1(\mathbb{G}) \cong A*_{\alpha_{\bar{e}}\alpha_e^{-1}}$ for some group A.

Exercise 5: Let $(\mathcal{G}, \Gamma, \Lambda)$ be a graph of groups, where Γ is finite. Determine the fundamental group of $(\mathcal{G}, \Gamma, \Lambda)$ from the vertex and edge groups just by using free products with amalgamation and HNN extensions.

Only four of these five exercises are mandatory.

* This is also a written exercise.

due: 20th December 2022