Fachbereich Mathematik
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WS 2022/23
6th December 2022

Exercises for Geometric Group Theory

Sheet 8

Exercise 1: Let G be a group and let H be a normal subgroup of G such that H and G/H are noetherian. Prove that G is noetherian.

Exercise 2*: Let T be an infinite tree that is not a double ray. Let G be a group acting on T with the following properties:

(1) the action on the vertices has exactly two orbits;

(2) the action on the edges is transitive and free.

Prove that $G = G_v * G_w$ for adjacent vertices $v, w \in V(T)$.

Exercise 3: Let $\Gamma = (V, E)$ be a connected graph with involution. Prove that every equivalence class of $\sim$ contains exactly one spikeless walk.

Exercise 4: Let F be a free group. Prove that F is isomorphic to the fundamental group of some graph with involution.

Exercise 5: Let Γ be a connected graph and let $\Delta \subseteq \Gamma$ be a subgraph. Show the following:

$$\pi_1(\Gamma) \Big/ \left(\langle \pi_1(\Delta) \rangle^{\trianglelefteq} \right) \cong \pi_1\left(\Gamma / E(\Delta)\right).$$

Only four of these five exercises are mandatory.

* This is also a written exercise.

due: 13th December 2022