Fachbereich Mathematik
Matthias Hamann

WS 2022/23
29th November 2022

Exercises for Geometric Group Theory

Sheet 7

Exercise 1: Let G be a group. Prove or disprove the following statement. There exists a tree T and a hyperbolic action of G on T if and only if G has a free subgroup of rank 2.

Exercise 2: Let the group G act on a tree T without inversion. Prove or disprove the following statement. Every $g \in G$ with infinite order is hyperbolic.

Exercise 3*: Let T be a k-regular tree for some $k \in \mathbb{N}$ with $k \geq 3$. Construct a group with a parabolic action on T.

Exercise 4: For each case in Theorem 4.1.7 construct a locally finite tree T such that T and its automorphism group satisfy this situation, i. e. such that the action is (1) elliptic, (2) hyperbolic, (3) cyclic, (4) dihedral or (5) parabolic.

Exercise 5: Let the group G act on X. Let $H_1, H_2 \leq G$ with $|H_1| \geq 3$. Let A, B be two non-empty disjoint subsets of X. We assume $gB \subseteq A$ for all $g \in H_1$ with $g \neq 1$ and $gA \subseteq B$ for all $g \in H_2$ with $g \neq 1$. Prove that the subgroup of G generated by H_1 and H_2 is isomorphic to $H_1 * H_2$.

Only four of these five exercises are mandatory.

* This is also a written exercise.

due: 6th December 2022