Fachbereich Mathematik
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WS 2022/23
22nd November 2022

Exercises for Geometric Group Theory

Sheet 6

Exercise 1: Let Γ be a locally finite graph. Prove one of the following.

(1) If Γ is infinite, then it contains an end.

(2) Every end contains a **geodesic** ray, i. e. a ray all of whose subpaths are geodesic.

Exercise 2: Let G be a finitely generated group and let S be a finite generating set of G. Show the following.

(1) For all $r, r' \in \mathbb{N}$ we have

$$\beta_{G,S}(r + r') \leq \beta_{G,S}(r) \cdot \beta_{G,S}(r').$$

(2) Let F be a free group with free generating set S. For all $r \in \mathbb{N}$ we have

$$\beta_{G,S} \leq \beta_{F,S}.$$

Exercise 3: Prove the following.

(1) Domination of (generalised) growth functions is a quasiorder.

(2) Equivalence of (generalised) growth functions is an equivalence relation.

Exercise 4*:

(1) Show that $\mathbb{Z}^n$ has polynomial growth for all $n \in \mathbb{N}$.

(2) Show that finitely generated free groups have exponential growth.

Exercise 5: Let G be a finitely generated infinite group with the following properties.

(1) $G \sim_{QI} G \times G$

(2) G does not have exponential growth.

Show that G has intermediate growth.

Only four of these five exercises are mandatory.

* This is also a written exercise.

due: 29th November 2022