Fachbereich Mathematik
Matthias Hamann

WS 2022/23
24th January 2023

Exercises for Geometric Group Theory

Sheet 13

Exercise 1: Let Γ be a δ-hyperbolic graph and let $c > 8\delta$. Let P be a c*-locally geodesic path* that is for all $x, y \in V(P)$ with $d(x, y) \leq c$ we have

$$d_P(x, y) = d(x, y).$$

Let Q be a geodesic path with the same end vertices as P. Show that there exists a constant κ that depends only on δ and c such that $d(z, Q) \leq \kappa$ for all $z \in V(P)$ and $d(z, P) \leq \kappa$ for all $z \in V(Q)$.

Exercise 2: Complete the proof of Proposition 5.2.2, that is, prove the inequality

$$d_S(1, g_i h^n)) \geq d_S(1, g_i) + n \cdot d_S(1, h)$$

for all $n \in \mathbb{N}$.

Exercise 3: Let G be a free products with amalgamation $A *_C B$ over a finite group C. Show that G is hyperbolic if and only if A and B are hyperbolic.

Exercise 4: Let Γ be a δ-hyperbolic graph, let $r \in \mathbb{N}$ and let $x, y \in V(\Gamma)$ with $d(x, y) > 8r + 4\delta + 1$. Let $u \in B_r(x)$ and $v \in B_r(y)$. Let P be a shortest x-y path and let Q be a shortest u-v path. Show that the midpoint of P has distance at most $4\delta + 1$ to a vertex on Q which lies at most r away from the midpoint of Q.

(A *midpoint* of a path is either its central vertex or a vertex incident with its central edge, depending on the parity of the length of the path.)

Exercise 5: Show that $\mathbb{Z}^2$ has only finitely many cones with respect to its standard generating set.

Only four of these five exercises are mandatory.

due: 31st January 2023