Fachbereich Mathematik
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WS 2022/23
17th January 2023

Exercises for Geometric Group Theory

Sheet 12

Exercise 1: Let $(T, \bar{\cdot}, \prec)$ be a tree set. Show that

$$[t_1, t_2] := \{t \in T \mid t_1 \prec t \prec t_2\}$$

is a total order.

Exercise 2: Let $(T, \bar{\cdot}, \prec)$ be a tree set. Show that

$$t_1 \sim t_2 :\leftrightarrow t_1 = t_2 \vee (t_1 \prec \bar{t}_2 \wedge \neg \exists t \in T \colon t_1 \prec t \prec \bar{t}_2)$$

is an equivalence relation.

Exercise 3: Let $(T, \bar{\cdot}, \prec)$ be a tree set. Show that $(T/\sim, T)$ is a tree.

Exercise 4*: Show that a finitely generated group is accessible if and only if it is the fundamental group of a finite graph of groups whose edge groups are finite and whose vertex groups are finitely generated and have at most one end.

Exercise 5: Let Γ be a hyperbolic graph. Show that there exists $\lambda \geq 0$ such that for all $x, y \in V(\Gamma)$ and every two geodesic x-y paths $x_0 \dots x_n$ and $y_0 \dots y_n$ in Γ we have $d(x_i, y_i) \leq \lambda$.

Only four of these five exercises are mandatory.

* This is also a written exercise.

due: 24th January 2023