Fachbereich Mathematik
Matthias Hamann

WS 2022/23
20th December 2022

Exercises for Geometric Group Theory

Sheet 10

Exercise 1: Let $\mathbb{G} = (\mathcal{G}, \Gamma, \Lambda)$ be a graph of groups with finite Γ such that all vertex groups are finitely generated. Prove that $\pi_1(\mathbb{G})$ is finitely generated.

Exercise 2: Let $\mathbb{G} = (\mathcal{G}, \Gamma, \Lambda)$ be a minimal graph of groups such that $E(\Gamma) \neq \emptyset$. Show that $\pi_1(\mathbb{G})$ is a proper free product with amalgamation or HNN extension.

Exercise 3: Prove Lemma 4.4.4.

Exercise 4: Prove Lemma 4.4.5.

Exercise 5*: Show that a graph of groups $\mathbb{G} = (\mathcal{G}, \Gamma, \Lambda)$ is minimal if and only if there is no proper subgraph Δ of Γ such that the restriction $\mathbb{G}|_\Delta$ of $\mathbb{G}$ to Δ leads to $\pi_1(\mathbb{G}) = \pi_1(\mathbb{G}|_\Delta)$.

Exercise 6: Let $\mathbb{G} = (\mathcal{G}, \Gamma, \Lambda)$ be a graph of groups. A vertex $x \in V(\Gamma)$ is called *terminal* if there exists a unique edge with x as initial vertex. Prove the following.

(1) If $\mathbb{G}$ is minimal, then there exists no terminal vertex.

(2) If $\mathbb{G}$ has no terminal vertex and Γ is finite, then $\mathbb{G}$ is minimal.

Exercise 7: Show that removing an inessential pair from a graph of groups as in Remark 4.5.3 leads to an isomorphic fundamental group.

Only four of these seven exercises are mandatory.

* This is also a written exercise.

due: 10th January 2023