ABSTRACT

Recent advances in separation of single channel signals have been achieved with a combination of independent subspace analysis (ISA) and time-frequency transforms. A basic component in these strategies is to reduce the dimension of time-frequency data prior to the application of independent component analysis (ICA). In this context, a standard strategy for dimensionality reduction is PCA, but other nonlinear methods have also been proposed over the last years. In this paper, we present a qualitative comparison of different dimensionality reduction methods in the context of single channel signal separation, together with the usage of modern clustering algorithms for grouping components obtained when using independent subspace analysis.

Keywords—Dimensionality Reduction, Wavelet, STFT, Signal Separation, Independent Subspace Analysis, Persistent Homology, Clustering Algorithms

1. INTRODUCTION

Signal separation is a central topic in many engineering fields requiring new mathematical tools due to the increasing complexity of modern application problems. In the last decade, different approaches have been proposed for dealing with the blind source separation of single channel signals. A strategy that has been proposed in [2, 3] integrates the well known method of independent component analysis (ICA) with time-frequency transforms. A crucial step in this framework is to reduce the dimensionality of the data prior to the application of ICA.

In recent developments of data analysis new strategies for dimensionality reduction have been inspired by geometrical and topological concepts [4]. New algorithms based on differential geometry are Whitney embedding based methods, Isomap, LTSA, Laplacian eigenmaps, Riemannian normal coordinates, to mention but a few. In parallel developments, probabilistic conditions and numerical algorithms (e.g. persistent homology) have provided new tools for reconstructing the homology of a manifold \( M \subset \mathbb{R}^n \) from a finite dataset \( X = \{x_i\}_{i=1}^m \subset M \). These techniques have also been applied in signal separation problems and clustering tasks.

The objective of this paper is to use these new developments in the framework of time-frequency independent subspace analysis. Our main contributions are the evaluation of different dimensionality reduction techniques, together with cluster techniques for improving the quality of the signal separation. The outline of this paper is as follows. In Section 2, we describe the basic elements of time-frequency independent subspace analysis. In Section 3, we describe the basic elements of our modulation map framework based on dimensionality reduction. An important component of our work are efficient interpolation techniques based on radial basis functions that we discuss in Section 4.

2. TIME-FREQUENCY ISA

In our problem, we consider a bandlimited signal \( f \in L^2(\mathbb{R}) \) and a segmentation of its domain in such a way that small consecutive signal patches are analyzed, as routinely performed in STFT or wavelet analysis. For instance, the set of signal patches can be defined as a dataset

\[
X_f = \{x^f_i\}_{i=1}^m, \quad x^f_i = (f(t_{k(i-1)+j}))_{j=0}^{n-1} \in \mathbb{R}^n,
\]

for \( k \in \mathbb{N} \) a fixed hop-size. Here, the regular sampling grid \( \{t_i\}_{i=0}^{km-k+n-1} \subset \mathbb{R} \) is constructed when considering the Nyquist-Shannon theorem for \( f \).

The fundamental problem of signal separation has been described in different applications. A particular example are cocktail party effect problems, where \( f = g + h \) is a mixture of two signals \( g \) and \( h \), and the objective is to separate \( g \) and \( h \) from \( f \).

Our concrete acoustical example is a one-channel signal \( f \) composed of two different instruments (represented by \( g \) and \( h \)). It is reasonable to obtain sample patches \( x_g \in X_g \) and \( x_h \in X_h \), but due to their complex frequency characteristics, an accurate separation of \( f \), specially when \( g \) and \( h \) are played simultaneously, is a challenging problem. In the particular case of noise reduction, power spectral subtraction is a fundamental strategy which removes the noise signal \( g \) from \( f = g + h \) by subtracting the frequency content \( |\hat{f}_k| - |\hat{h}_k| \) at each frequency bin \( k \) [7]. A basic hypothesis is that the noise and clear signal vectors are orthogonal to each other. But this assumption is usually wrong, and a generalized approach takes into account a more accurate geometrical relation between the noise and signal vectors [7]. In our framework we use this generalized scenario but considering point cloud data structures instead of single frequency bins.

Given a signal \( s = \sum s_i \) containing a mixture of signals \( s_i \), the accurate extraction of signals is the denominated signal sep-
ation problem. There are different variations including cock-
tail party effect problem,

\[
X_g \xrightarrow{T} T(X_g) \quad T(X_g) \cup T(X_h) \xrightarrow{R} \Omega_{p\cup h}
\]

\[
X_h \xrightarrow{T} T(X_h)
\]

Independent subspace analysis (ISA) is a natural generaliza-
tion of independent component analysis (ICA). Recall than in
ICA, we have a

3. DIMENSIONALITY REDUCTION AND SIGNAL
TRANSFORMS

Due to the high dimensionality of the time-frequency data, it is
of interest to work with analysis techniques that combine sig-
nal processing transforms with dimensionality reduction meth-
ods. In this case, the basic objects are the manifold \(M\), the
data samples \(X = \{x_i\}_{i=1}^m \subset \mathbb{R}^n\), and a diffeomor-
phism \(A : \Omega \rightarrow M\), where \(\Omega\) is the low-dimensional copy of
\(M\) to be reconstructed via dimensionality reduction. Here, the
only algorithmic input is the dataset \(X\), but with the assump-
tion that we can reconstruct topological information of \(M\) with \(X\)
(see for instance [5]). The other basic object in our scheme is a
signal processing map \(T : M \rightarrow M_T\), which may be based on
Fourier analysis, wavelet transforms, or convolution filters,
together with the resulting set \(M_T := \{T(p), p \in M\}\) of trans-
formed data. The following diagram shows the basic situation.

\[
\begin{align*}
\Omega \subset \mathbb{R}^d & \xrightarrow{A} X \subset M \subset \mathbb{R}^n \\
\Omega' \subset \mathbb{R}^d & \xleftarrow{R} T(X) \subset M_T \subset \mathbb{R}^d
\end{align*}
\]

The main objective is to find an approximation of \(\Omega\), denoted
\(\Omega' = R(M_T)\), by using a suitable dimensionality reduction map \(R\). Some properties of \(\Omega\) and \(\Omega'\) may differ depending
on the dimensionality reduction technique, but the target is to
construct \(\Omega'\), so that geometrical and topological properties of
\(\Omega\) are recovered. In Section ??, we use a particular modulation
map \(A\) and we study the geometrical effects being incurred by
several dimensionality reduction maps \(R : M \rightarrow \Omega'\).

4. RADIAL BASIS FUNCTION INTERPOLATION

An important ingredient in our framework are radial basis
functions (RBF) and their interpolation methods for high-
dimensional data. As previously described in our framework
(Section ??), our usage of dimensionality reduction requires to
map data between high and low dimensional spaces. Some
dimensionality reduction methods have intrinsic interpola-
tion strategies, but in general, the reconstruction of high-
dimensional data from the low-dimensional representation is
a non trivial problem. In order to consider a flexible frame-
work that embraces different reduction methods, we consider
the multi-purpose features of RBFs as already suggested, for
instance, in [1].

The inputs of the RBF interpolation methods are the datasets
\(X = \{x_i\}_{i=1}^m \subset \mathbb{R}^n\) and \(Y = \{y_i\}_{i=1}^m \subset \mathbb{R}^d\). The RBF inter-
Polant requires a family of centers \(\{c_j\}_{j=1}^N\) (simply chosen ran-
domly from the datasets in our experiments), and can be written
as:

\[
y_k = w_0 + \sum_{j=1}^N w_j \phi(||x_k - c_j||), \quad k = 1, \ldots, m,
\]
or in matrix form, this can be described as:

\[
Y = W \Phi^T,
\]

with the \(m \times (N + 1)\) matrix \(\Phi = (1_m, \Phi_1, \ldots, \Phi_N), (\Phi_j)_k = \phi(||x_k - c_j||), \) the \(d \times (N + 1)\) coefficient matrix \(W =
(w_0, \ldots, w_N)\), and the \(d \times m\) matrix \(Y = (y_1, \ldots, y_m)\).
A solution can be constructed with the pseudo-inverse \(\Phi^\dagger: \Phi^\dagger W^T = \Phi^\dagger Y^T\). In our computational experiment we use the
Gaussian RBF \(\phi(r) = \exp(-r^2/\alpha)\), for some fixed \(\alpha > 0\).

5. CLUSTERING TECHNIQUES

6. COMPUTATIONAL EXPERIMENTS

![Image](321x312 to 453x411)

**Fig. 1.** (a) The PCA 3D projection of the frequency content of \(M\). (b) The Isomap 3D projection of the frequency content of \(M\).

7. REFERENCES

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Fig. 2. (a) The PCA 3D projection of the frequency content of $\mathcal{M}$. (b) The Isomap 3D projection of the frequency content of $\mathcal{M}$.
