# DIMENSIONALITY REDUCTION METHODS IN INDEPENDENT SUBSPACE ANALYSIS FOR SIGNAL DETECTION

*Mijail Guillemard, Armin Iske, and Sara Krause-Solberg*

Department of Mathematics, University of Hamburg, Germany
{guillemard,iske}@math.uni-hamburg.de, sara.krause-solberg@hamburg.de

## ABSTRACT

In the last few years an important family of methods for single-channel signal separation has been developed using tools from time-frequency analysis. Given a mixture of signals $f = \sum_i f_i$, the task is to estimate the components $f_i$ using specific assumptions on their time-frequency or statistical characteristics. A well-known strategy, termed *independent subspace analysis* (ISA), is to reduce the embedding dimension of the time-frequency representation of $f$, prior to the application of independent component analysis (ICA). In these methods, a standard strategy for dimensionality reduction is (the linear) principal component analysis (PCA), but also nonlinear methods have recently been proposed. In this paper, we compare different dimensionality reduction methods for single channel signal separation in the context of ISA. Our focus is on signals with transitory components, and the objective is to detect the locations in time where each individual signal $f_i$ is activated.

***Keywords***— Dimensionality reduction, independent subspace analysis, signal detection and separation, wavelets, STFT.

## 1. INTRODUCTION

Signal separation is a crucial task in many application fields, and its modern development depends on experimental breakthroughs supported by a correct understanding of the underlying mathematical framework. In the last decade, several approaches have been proposed for the problem of blind source separation of single channel signals. A fundamental strategy proposed in [1,2] combines independent component analysis (ICA) methods with time-frequency transforms. These ideas have been extended by considering other types of matrix decompositions in addition to the statistically oriented strategy of ICA. In particular, methods using non-negative matrix factorization techniques have gained significant attention in recent years.

A crucial step in many of these strategies is to reduce the dimension of the Euclidean space where the time frequency representation is embedded. New methods for dimensionality reduction of point cloud data $X = \{x_i\}_{i=1}^m \subset \mathbb{R}^n$ have actively been developed using geometrical and topological concepts [8]. Novel algorithms based on concepts from differential geometry are Whitney embedding based methods, isomap, local tangent space alignment (LTSA), Laplacian eigenmaps, Riemannian normal coordinates (RNC), to mention but a few.

The objective of this paper is to evaluate the usage of new dimensionality reduction tools in signal detection and separation algorithms. Recent developments on this topic were presented in [2], but further investigations on the signal processing and mathematical framework of these algorithms are essentially required. This paper discusses various important aspects concerning the application of dimensionality reduction methods in the context of ISA. In particular, we focus on the signal detection problem in a complex mixture of transitory acoustic sounds. A better mathematical understanding of these procedures, and additional empirical insights, are fundamental for improving current strategies and designing new methods for signal separation.

The outline of this paper is as follows. In Section 2, we discuss the utility of dimensionality reduction methods in combination with signal processing tools, with a particular focus on the analysis of time-frequency data. We also present some basic ideas on ICA and dimensionality reduction strategies. In Section 3, we briefly review current techniques proposed in the literature for independent subspace analysis. Finally, in Section 4, we present computational experiments illustrating the signal detection capabilities of the ISA and dimensionality reduction framework using two different methods, the linear PCA method and nonlinear Laplacian eigenmaps.

## 2. DIMENSIONALITY REDUCTION AND SIGNAL PROCESSING

A basic characteristic of short term Fourier transforms (STFT), is the high dimensionality of the Euclidean space where their time-frequency data is embedded. In this context, for many applications a preprocessing step using dimensionality reduction methods for time-frequency transforms can potentially improve the quality of the data analysis. These preprocessing algorithms are at the heart of many modern signal separation methods and, in particular, they are fundamental for the ISA methods we discuss in this paper. We first formalize these ideas by describing a general framework with a close integration of signal processing tools with dimensionality reduction maps (see also [4,5]).

We consider a band-limited signal $f \in L^2(\mathbb{R})$ and a segmentation of its domain in such a way that small consecutive

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signal patches are analyzed, as routinely performed in STFT or wavelet analysis. For instance, the set of signal patches $X_f$ can be defined as a dataset of vectors in $\mathbb{R}^n$ , derived by drawing $m$ samples from a signal $f$:

$$X_f = \{x_i\}_{i=1}^m \subset \mathbb{R}^n, \quad x_i = (f(t_{k(i-1)+j}))_{j=0}^{n-1} \in \mathbb{R}^n,$$

for $k \in \mathbb{N}$ a fixed hop-size. Here, the regular sampling grid $\{t_\ell\}_{\ell=0}^{km-k+n-1} \subset \mathbb{R}$ is constructed when considering the Nyquist-Shannon theorem for $f$. Now, a fundamental and reasonable assumption is that $X_f$ lies in $\mathcal{M}$, a (low-dimensional) *space* (manifold or topological space e.g. CW-complex, simplicial complex) embedded in the high dimensional space $\mathbb{R}^n$. Therefore, we have $X_f \subset \mathcal{M} \subset \mathbb{R}^n$ with $p := \dim(\mathcal{M}) \ll n$. The construction of time-frequency data can be described as the application of a map $T : \mathcal{M} \supset X_f \to T(X_f) \subset \mathcal{M}_T$, where $\mathcal{M}_T := T(\mathcal{M})$, and $T(x_i)$ is the signal transformation of $x_i$ (Fourier transform, wavelet, etc). An additional key concept is the consideration of an *ideal model*, $\Omega$, representing $\mathcal{M}$, being embedded in a low-dimensional space $\mathbb{R}^d$ (with $d \ll n$), together with a homeomorphism (or even an isometry) $\mathcal{A} : \mathbb{R}^d \supset \Omega \to \mathcal{M} \subset \mathbb{R}^n$. The space $\Omega$ represents an ideal representation of $\mathcal{M}$ that would allow convenient analysis procedures in a low-dimensional environment. For instance, in the case where $\mathcal{M}$ is the well-known *Swiss roll dataset*, $\Omega$ is a rectangle. However, in practice, we can only try to approximate $\Omega$ with a dimensionality reduction map $P : \mathbb{R}^n \supset \mathcal{M}_T \to \Omega' \subset \mathbb{R}^d$, where $\Omega'$ is a homeomorphic copy of $\Omega$. The following diagram shows the basic framework.

$$\begin{tikzcd}
\mathbb{R}^d \supset \Omega \arrow[r, "\mathcal{A}"] & \mathcal{M} \supset X_f \subset \mathbb{R}^n \arrow[d, "T"] \\
\mathbb{R}^d \supset \Omega' & \mathcal{M}_T \supset T(X_f) \subset \mathbb{R}^n \arrow[l, "P"']
\end{tikzcd}$$

We recall that there is a well-known framework for studying properties of such sets $X_f$ in the context of nonlinear time series and dynamical systems (see e.g. [7]). But in our framework, we are additionally considering a close interaction with signal processing transforms $T$, together with specialized dimensionality reduction techniques $P$.

We will now describe some general information on dimensionality reduction methods $P : \mathbb{R}^n \supset \mathcal{M}_T \to \Omega \subset \mathbb{R}^d$, and in particular, in Section 4 we use the well-known linear PCA method, and modern nonlinear strategies such as Laplacian eigenmaps.

**Nonlinear Dimensionality Reduction**

The high dimensionality property of many real-life signals is a fundamental problem for data analysis algorithms. Analyzing and interpreting high-dimensional datasets can be a mathematical and computational challenge for traditional statistical and linear methods. In many cases, a dataset has an intrinsic low-dimensional structure that is relevant for understanding its underlying characteristics. Low-dimensional representations of datasets are also easier to operate for classification, visualization or compression purposes. Efficient dimensionality reduction algorithms are therefore crucial methods for finding useful low-dimensional embeddings of datasets. This lower dimension should ideally correspond to the intrinsic dimensionality of the data, and different strategies are available for estimating these dimensions [10].

There are two major types of dimensionality reduction methods: linear and nonlinear ones. In this context, linearity refers to the idea that each element of the dataset is a linear combination of the other data points, see [3]. Nonlinear techniques are usually based on the construction of a low-dimensional representation of a datasets, with aiming at [10]:

1. preservation of global properties;
2. preservation of local properties;
3. composition of linear techniques.

**Independent Component Analysis (ICA)**

The ICA algorithm is a separation algorithm based on statistical principles for unmixing a linear combination of signals. In itself, ICA is not a dimensionality reduction method, but it is frequently used in combination with classical dimensionality reduction methods such as PCA. The input of the ICA algorithm is the point cloud data $X = \{x_i\}_{i=1}^m \subset \mathbb{R}^n$ written in matrix form as $X = (x_1 \dots x_m) \in \mathbb{R}^{n\times m}$. The objective is to find a matrix of source signals $S = (s_1 \dots s_m) \in \mathbb{R}^{n\times m}$, assuming a linear dependence between $X$ and $S$. By denoting the *mixing matrix* as $W \in \mathbb{R}^{n\times n}$, this can be expressed as:

$$X = WS, \quad X, S \in \mathbb{R}^{n\times m}, W \in \mathbb{R}^{n\times n}.$$

In this equation, the mixing matrix $W$ and the source signals $S$ are unknown variables, and ICA estimates these matrices using some assumptions on the statistical independence of the signals $\{s_i\}_{i=1}^n$. The general strategy uses the following measure for a set of random variables $Y = \{y_i\}_{i=1}^n$:

$$I(Y) = D(P_Y, \prod_i P_{Y_i}), \quad D(p, q) = \int_{\mathbb{R}} p(x) \log\left(\frac{p(x)}{q(x)}\right) dx.$$

The measure $I$ allows us to compute the degree of statistical independence by comparing the joint distribution $P_Y$, and the marginal distributions $P_{Y_i}$. The comparison function $D$, used in the measure $I$, is the *Kullback-Leibler distance*, also known as *relative entropy*. This allows us to formulate the ICA algorithm as an optimization problem, where the solution space is the *general linear group*, defined as the set of $n \times n$ invertible matrices, $\mathrm{GL}(n, \mathbb{R}) = \{A \in \mathbb{R}^{n\times n},\ \det(A) \neq 0\}$, with $p(A) := I(A^{-1}X)$:

$$W = \underset{A\in \mathrm{GL}(n,\mathbb{R})}{\operatorname{argmin}}\ p(A).$$

## 3. ISA AND TIME-FREQUENCY DATA

With the previous background on dimensionality reduction and unmixing methods based on ICA, we now describe ISA as an important strategy for single-channel signal separation. We recast the concepts with following along the lines of [1, 2, 9, 11].

The original meaning of the term *independent subspace analysis* (ISA) is related to a generalization of independent component analysis by considering a multidimensional version of ICA. In very general terms, the main idea is to group the source vectors $\{s_i\}$ in subfamilies or linear subspaces, where the elements within each subgroup are statistically dependent, but elements of different groups are statistically independent (see e.g. [6]). Due to the work of Casey and Westner [1], the term ISA has frequently been used to denote methods using ICA to decompose power spectrograms of single channel signals for sound separation (see [11]).

The concept of ISA for single channel signal separation consist of decomposing a signal $f = \sum f_i$, by applying ICA to a dimensionality reduced representation of the power spectrogram. More precisely, given the function $f$, we compute the power spectrogram by considering $T(x_i)$ as the magnitude of the discrete Fourier transform FT of each element of the point cloud data $X_f = \{x_i\}_{i=1}^m$. Namely, $T(x_i)_k = \|\mathrm{FT}(x_i)_k\|$, for $k = 1, \ldots, m$. The following step is to use the power spectrogram $T(X_f)$ for constructing its low-dimensional representation $P(T(X_f))$ using a linear or nonlinear method $P$ (e.g. PCA, Laplacian eigenmaps, LTSA, etc). We then apply ICA to the dataset $P(T(X_f))$ for unmixing the resulting signals, to obtain estimations of the components $f_i$. We briefly explain these steps as follows (cf. [1, 9, 11]).

1- Construct a dimensionality reduced power spectrogram using principal component analysis, (i.e. $P = \mathrm{PCA}$),

$$X_f \xrightarrow{T} T(X_f) \xrightarrow{P} P(T(X_f))$$

2- Unmix the resulting matrix $P(T(X_f))$ using ICA,

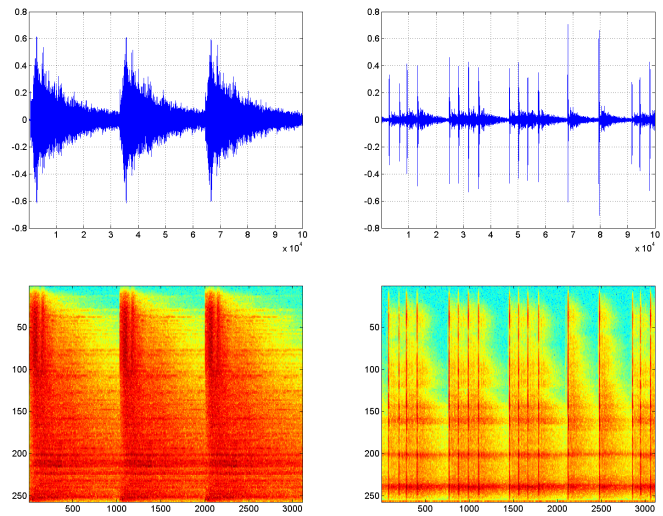
$$P(T(X_f)) \xrightarrow{\text{ICA}} \{\tilde{f}_i\}$$

3- Group different components $\{\tilde{f}_i\}$ to estimate the signals $f_i$ in $f = \sum_i f_i$.

In order to reconstruct the spectrogram for each $f_i$, we apply the inverse short term Fourier transform with the phase information of the original mixture signal $f$ (see [1, 9, 11] for details). We consider using different dimensionality reduction methods $P$, with focussing on Laplacian eigenmaps and isomap. We finally remark that in recent developments new types of unmixing methods have been used in step 2 for replacing the statistically oriented ICA strategy. In particular, non-negative matrix factorizations have gained increasing attention (see [11]).

## 4. COMPUTATIONAL EXPERIMENTS

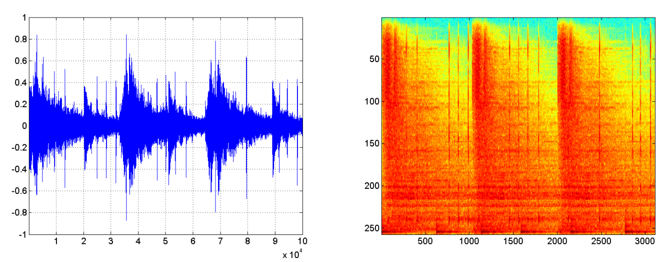
To compare the signal detection quality for different dimensionality reduction methods, we construct a mixture of acoustic transient signals. We consider $f = f_1 + f_2$, where $f_1$ is a sequence of *castanets* and $f_2$ a *cymbal* signal, as depicted in Figure 1.



**(a)** **(b)**

**Fig. 1**. (a) Cymbal signal $f_1$; (b) castanets signal $f_2$.

The combination $f = f_1 + f_2$ of the cymbal and castanets is depicted in Figure 2. The power spectrogram in this figure represents the point cloud data $T(X_f)$, where each column represents the FT of a segment $x_i$ of the signal $f$. Due to the complex frequency characteristics of $f$, identifying and extracting the castanets signals is a very challenging task.



**(a)** **(b)**

**Fig. 2**. Signal $f = f_1 + f_2$ and its power spectrogram $T(X_f)$.

In Figures 3-5, we present detection results for identifying the positions of the castanets and cymbal signals. To this end, we use the dimensionality reduction methods $P = \mathrm{PCA}$, $P = \mathrm{LE}$ (Laplacian eigenmaps), and $P = \mathrm{isomap}$. In the case of the two nonlinear methods ($P = \mathrm{LE}$, $P = \mathrm{isomap}$), a suitable nearest neighbor parameter $k$ can be selected [1]. We have manually identified some values for $k$ that we display in Figures 4,5.

[1] Code available at `www.math.uni-hamburg.de/home/guillemard/`

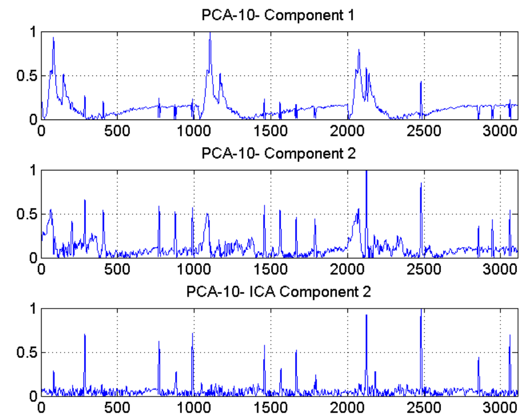



**Fig. 3**. **PCA.** (a) cymbal (b) castanets (c) castanets (ICA).

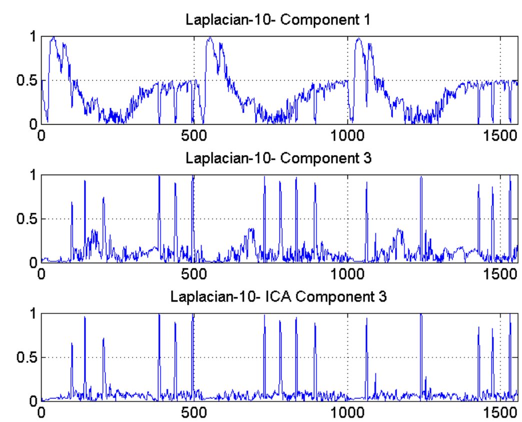



**Fig. 4**. **LE.** (a) cymbal (b) castanets (c) castanets (ICA).

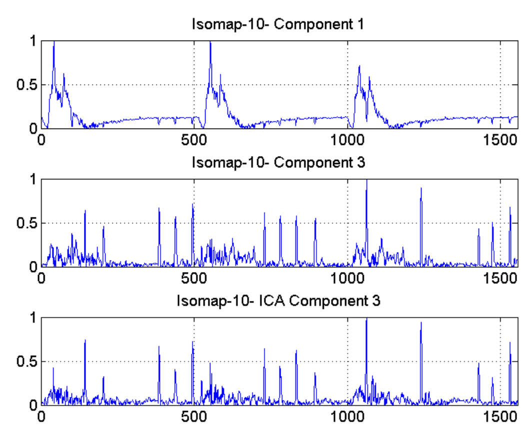



**Fig. 5**. **Isomap.** (a) cymbal (b) castanets (c) castanets (ICA).

In Figures 3-5, (a) shows a component $f_i$ that matches the cymbal signal, and the two plots (b) and (c) correspond to the components related to the castanet signal. Moreover, plots (a) and (b) are obtained from the detection algorithm without the ICA procedure, whereas in plot (c), the ICA procedure is activated. Note that the unmixing ICA algorithm slightly improves the quality of the castanets detection by decreasing some background signal in the case of PCA and Laplacian eigenmaps. In conclusion, these dimensional reduction strategies are able to detect the relevant signals for this particular example fairly well, but further steps are required to improve the detection quality and the analysis of resynthesis algorithms.

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