# Curvature Analysis of Frequency Modulated Manifolds in Dimensionality Reduction

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**Abstract**

Recent advances in the analysis of high-dimensional signal data have triggered an increasing interest in geometry-based methods for nonlinear dimensionality reduction (NDR). In many applications, high-dimensional datasets typically contain redundant information, and NDR methods are important for an efficient analysis of their properties. During the last few years, concepts from differential geometry were used to create a whole new range of NDR methods. In the construction of such geometry-based strategies, a natural question is to understand their interaction with classical and modern signal processing tools (convolution transforms, Fourier analysis, wavelet functions). In particular, an important task is the analysis of the incurred geometrical deformation when applying signal transforms to the elements of a dataset. In this project, we propose the concept of frequency modulation maps and modulation manifolds for the construction of particular datasets relevant in signal processing and NDR. Moreover, we design a numerical algorithm for analyzing geometrical properties of the modulation manifolds, with a particular focus on their scalar curvature. Finally, in two numerical examples, we apply the resulting geometry-based analysis algorithm to two model problems where we present geometrical and topological effects of relevance in manifold learning.

*Keywords:* Nonlinear dimensionality reduction, manifold learning, signal processing, Fourier and wavelet analysis, numerical differential geometry.

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