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Exercise sheet 3

Submit your solutions in the exercise group on 2011-May-02!

Exercise 9: Complex tori. (*10 points*)

Let $(b_1, \ldots, b_{2n})$ be a real basis of $\mathbb{C}^n$, let Γ be the additive subgroup of $\mathbb{C}^n$ generated by $b_1, \ldots, b_{2n}$.

a. Prove that Γ acts by translations freely and properly discontinuously on $\mathbb{C}^n$.

b. Prove that $\mathbb{C}^n/\Gamma$ is diffeomorphic to the real torus $T^{2n} := \mathbb{R}^{2n}/\mathbb{Z}^{2n}$.

Exercise 10: Hopf manifolds. (*10 points*)

Let $n \in \mathbb{N}$, let $\lambda \in \mathbb{C}^\times = \mathbb{C}\backslash\{0\}$ satisfy $|\lambda| \neq 1$. The multiplicative subgroup $\langle\lambda\rangle$ of $\mathbb{C}^\times$ generated by λ acts on $\mathbb{C}^n\backslash\{0\}$ by multiplication.

a. Prove that $(\mathbb{C}^n\backslash\{0\})/\langle\lambda\rangle$ is diffeomorphic to $S^{2n-1} \times S^1$.

b. Prove that there exists an element τ of the upper half-plane $\mathcal{H}$ such that $\mathbb{C}^\times/\langle\lambda\rangle$ is biholomorphic to $\mathbb{C}/\Gamma_\tau$, where Γ_τ denotes the additive subgroup of $\mathbb{C}$ generated by 1 and τ.

Exercise 11: Grassmannians. (*10 points*)

Let $n \in \mathbb{N}$, let $k \in \{0, \ldots, n\}$. Prove that $\mathrm{Gr}_k(\mathbb{C}^n)$ is biholomorphic to $\mathrm{Gr}_{n-k}(\mathbb{C}^n)$.

Exercise 12: Fine sheaves are soft. (*10 points*)

Let $\mathscr{F}$ be a fine sheaf of abelian groups on a manifold M. Prove that $\mathscr{F}$ is soft.