Lösungshinweise zu Arbeitsblatt 5

Ableitungen der Funktionen von Aufgabe 5.2

a) $f(x,y) = 100(y-x^2)^2 + (1-x)^2$ *(Rosenbrock-Funktion)*

$$\nabla f(x,y) = \begin{pmatrix} -400x(y-x^2) - 2(1-x) \\ 200(y-x^2) \end{pmatrix}$$

$$\nabla^2 f(x,y) = \begin{pmatrix} 1200x^2 - 400y + 2 & -400x \\ -400x & 200 \end{pmatrix}$$

b) $f(x,y) = x^2y^2 - 0.25(2x^2 - y^2) + 0.5(2 - x^2 - y^2)^2$

$$\nabla f(x,y) = \begin{pmatrix} 2x^3 + x(4y^2 - 5) \\ 2y^3 + y(4x^2 - 3.5) \end{pmatrix}$$

$$\nabla^2 f(x,y) = \begin{pmatrix} 6x^2 + 4y^2 - 5 & 8xy \\ 8xy & 4x^2 + 6y^2 - 3.5 \end{pmatrix}$$

c) $f(x,y) = x^4 + xy + (1+y)^2$

$$\nabla f(x,y) = \begin{pmatrix} 4x^3 + y \\ x + 2y + 2 \end{pmatrix} \qquad \nabla^2 f(x,y) = \begin{pmatrix} 12x^2 & 1 \\ 1 & 2 \end{pmatrix}$$

d) $f(x,y) = x^2 + 10y^2$

$$\nabla f(x,y) = \begin{pmatrix} 2x \\ 20y \end{pmatrix} \qquad \nabla^2 f(x,y) = \begin{pmatrix} 2 & 0 \\ 0 & 20 \end{pmatrix}$$

e) $f(x,y) = x^2y^2 + (x^2-1)^2$ (Aufgabe 1.2 a)

$$\nabla f(x,y) = \begin{pmatrix} 2x(2x^2 + y^2 - 2 \\ 2x^2y \end{pmatrix}$$

$$\nabla^2 f(x,y) = \begin{pmatrix} 12x^2 + 2y^2 - 4 & 4xy \\ 4xy & 2x^2 \end{pmatrix}$$